ADVANCED ASTROPHYSICS

This book develops the basic underlying physics required for a fuller, richer understanding of the science of astrophysics and the important astronomical phenomena it describes. The Cosmos manifests phenomena in which physics can appear in its most extreme, and therefore more insightful, forms. A proper understanding of phenomena such as black holes, quasars and extrasolar planets requires that we understand the physics that underlies all of astrophysics. Consequently, developing astrophysical concepts from fundamental physics has the potential to achieve two goals: to derive a better understanding of astrophysical phenomena from first principles and to illuminate the physics from which the astrophysics is developed. To that end, astrophysical topics are grouped according to the relevant areas of physics. The book is ideal as a text for graduate students and as a reference for established researchers.

The author obtained his PhD in 1984 from the University of Toronto where he earned the Royal Astronomical Society of Canada Gold Medal for academic excellence. After a brief postdoctoral stint at the University of British Columbia, he joined the faculty at the University of New Mexico where he pursued his interests in radio astronomy. He has been teaching for the past 17 years, earning an “excellence in teaching” award for the graduate courses on which this book is based. Dr. Duric has over 100 scientific publications and has authored and/or edited five books. In addition, he has developed a number of online classes, including a completely interactive, web-based freshman astronomy course. He is the recipient of the “Regent’s Fellowship”, the highest honour that UNM bestows on its faculty. His research has taken him around the world to over a dozen countries, accounting in part, for the global perspective that characterizes his book. Dr. Duric is a member of the American Astronomical Society and the Canadian Astronomical Society.
ADVANCED ASTROPHYSICS

NEB DURIC

University of New Mexico
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Astrophysics strives to describe the Universe through the application of fundamental physics. The Cosmos manifests phenomena in which the physics can appear in its most extreme, and therefore more insightful, forms. Consequently, developing astrophysical concepts from fundamental physics has the potential to achieve two goals: to derive a better understanding of astrophysical phenomena from first principles, and to illuminate the physics from which the astrophysics is developed. To that end, astrophysical topics are grouped, in this book, according to the relevant areas of physics. For example, the derivation of the laws of orbital motion, used in the detection of extrasolar planets, takes place in the classical mechanics part of the book while the derivation of transition rates for the $21\text{ cm}$ neutral hydrogen line, used to probe galaxy kinematics, is performed in the quantum mechanics part. The book could serve as a text for graduate students and as a reference for established researchers.

The content of this book is based on the material used by the author in support of advanced astrophysics courses taught at the University of New Mexico. The intended audience consists of graduate students and senior undergraduates pursuing degrees in physics and/or astrophysics. Perhaps the most directly relevant demographic is the combined Physics and Astronomy departments. These departments tend to emphasize the fundamental physics regardless of the research track pursued by the student. In many cases a separate astrophysics degree is not an option. In these departments (such as the author’s) all students must pass the same physics comprehensive examination. Consequently, students must be well prepared in fundamental physics both from the points of view of course work as well as research. In the latter case a strong physics foundation is very helpful in developing thesis topics to an acceptable level in a physics-dominated department. This book is specifically aimed at those departments.

The department of Physics and Astronomy at the University of New Mexico requires its graduate students to take the physics comprehensive exam. Courses
based on the material in this book have helped astrophysics and physics students prepare for these exams. I attribute this benefit to the fact that the astrophysical topics provide interesting and insightful manifestations of the fundamental physics, of which, the students previously may have had only a theoretical knowledge. I therefore expect the book to impact the physics as well as the astrophysics students in the mixed departments. I also expect that graduate students in physics-only and astronomy-only departments may choose to use this book to hone their research skills. Targeting junior/senior undergraduates is also possible in schools where the science curricula are robust.

Multi-disciplinary and cross-disciplinary investigations are playing an increasingly important role in scientific research. The cross-over of particle physicists into cosmology, and the establishment of the field of astroparticle physics, is just one manifestation of the growing overlap between physics and astrophysics disciplines. The emphasis on the linkage of fundamental physics and astrophysics makes this book potentially useful as a reference to physics and astrophysics researchers who wish to broaden their research base.

The author acknowledges the help of Dr. Rich Epstein (Los Alamos National Laboratory) who co-taught the first course in the series of courses that have led to the development of the material used in this book. The cooperation and help of the many students who have taken these courses has been instrumental in identifying many typos and inconsistencies in the course material. Finally, the author acknowledges the help and support of the department of physics and astronomy at UNM and the patience and encouragement of family and friends in this endeavor.