JET PROPULSION

A Simple Guide to the Aerodynamics and Thermodynamic Design and Performance of Jet Engines

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Part 1

Design of Engines for a New 600-seat Aircraft
CHAPTER 1 THE NEW LARGE AIRCRAFT — REQUIREMENTS AND BACKGROUND

1.0 INTRODUCTION

This chapter looks at some of the commercial requirements and background to the proposals to build a new civil aircraft capable of carrying about 600 people. The costs and risks of such a project are huge, but the profits might be large too. In explaining the requirements some of the units of measurement used are discussed. Design calculations in a company are likely to assume that the aircraft flies in the International Standard Atmosphere (or something very similar) and this assumption will be adopted throughout this book. The standard atmosphere is introduced and discussed towards the end of the chapter. The chapter ends with brief reference to recent concerns about environmental issues.

1.1 SOME COMMERCIAL BACKGROUND

In December 2000 Airbus formally announced the plans to go ahead with a new large aircraft, dubbed the A380, intended in its initial version to carry a full payload (with 555 passengers) for a range of up to 8150 nautical miles. First flight is intended to be in 2004 and entry into service in 2006. There are already plans afoot for heavier versions, carrying more than 555 passengers and for all-freight versions with a larger payload. In December 2000 Airbus Industrie had received enough orders to justify the expected cost of over $10 billion, with an expected break-even point with a sale of 250 aircraft. They forecast delivery of the 250th aircraft in 2011.

The large capital expenditure and the long payback period highlight the risks, for cost over-run, project delay or slow sales could undermine all these estimates. Boeing, who have until now dominated the large end of the market with the Boeing 747, offered an updated version, the 747X to compete with the A380. The Boeing 747-400, currently the largest civil aircraft, was introduced into service in 1989 but it is a derivative of the 747-100 which entered service in 1970. The 747-400 incorporated some aerodynamic improvements, including improvements to existing engines, but more radical redesign would be needed to take full advantage of the developments in aerodynamics and materials since 1970. Adopting these technology developments for the A380, together with new engines, should result in a substantially more cost-efficient aircraft with about a 15% reduction in seat-mile costs, compared

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1 The word civil is used in Britain where commercial would be used in the USA.
with the Boeing 747-400. At the end of March 2001 Boeing had not received a single order for their 747X and the project was formally put on hold, Boeing stating that there was not an adequate market for a very large aircraft. While many expect that the 747X will ultimately be cancelled, Boeing firmly deny this. Boeing’s intentions for new very large aircraft are not clear and it must be assumed that this is a topic of intense consideration within the company.

For several years Boeing and Airbus Industrie have separately and jointly discussed proposals for a much larger aircraft, with anywhere from about 600 to about 800 seats. All the proposals for very large aircraft have four engines hung from under the wing. However, at the same time as the announcement that the 747X was being postponed, Boeing announced a very different aircraft, unofficially dubbed the "sonic cruiser". This would cruise at a Mach number of at least 0.95 (whereas the 747X would have cruised at $M=0.85$) with a range of 9000 nautical miles but with only about 200 seats. The specification is still fluid at the time of writing. In any case this higher speed aircraft, which some in the industry believe will ultimately be unattractive on economic and environmental grounds, is not the subject of this book, though the topic of high-speed passenger carrying aircraft is returned to briefly in Chapter 19.

1.2 THE NEW LARGE AIRCRAFT

The first ten chapters of this book are concerned with a hypothetical New Large Aircraft (NLA) which bears a close resemblance to proposals put out by Airbus and by Boeing from around 1996. The final aircraft launched as the Airbus A380-100 in December 2000 differs in a number of ways from these and the A380-100 is compared with the hypothetical New Large Aircraft are compared in Table 1.1.

<table>
<thead>
<tr>
<th></th>
<th>New Large Aircraft</th>
<th>Airbus A380-100*</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of passengers</td>
<td>620</td>
<td>555</td>
</tr>
<tr>
<td>Range (nautical miles)</td>
<td>8000</td>
<td>8150</td>
</tr>
<tr>
<td>Payload at this range (tonne)</td>
<td>58.8</td>
<td>52.9</td>
</tr>
<tr>
<td>Max. take-off weight (tonne)</td>
<td>635.6</td>
<td>560.2</td>
</tr>
<tr>
<td>Empty weight (tonne)</td>
<td>298.7</td>
<td>274.9</td>
</tr>
<tr>
<td>Cruise Mach number</td>
<td>0.85</td>
<td>0.85</td>
</tr>
<tr>
<td>Initial cruise altitude (feet)</td>
<td>31000</td>
<td>35000</td>
</tr>
<tr>
<td>Cruise Lift/Drag</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>Wing area (metre$^2$)</td>
<td>790</td>
<td>845</td>
</tr>
</tbody>
</table>

* specifications as of 1 May 2001  
1 tonne = 2205 lb mass
The main differences between the hypothetical New Large Aircraft and the A380-100 are the range, weight and smaller wing area of the hypothetical aircraft. The wing area assumed is close to one that Airbus first proposed before increasing it in a series of steps over the last five or so years. The larger wing area allows future growth in aircraft weight, but also allows take-off and landing at lower speeds, thereby reducing noise nuisance. The differences are sufficiently small that the aim of the book, which is the understanding of the aerodynamic and thermodynamic constraints and decisions for the propulsion of a new large civil aircraft, are not compromised by retaining the numerical values for the original hypothetical new large aircraft.

The price of a new aircraft is a complex issue, depending on the level of fittings inside the aircraft and on the various discounts offered. It may be assumed that the catalogue price of an A380 will be of the order of $200 million, with the engines costing around $12 million each. The market is variously estimated to be between 1100 and 1300 very large aircraft over the next 20 years, Boeing suggesting a much smaller number. Airbus want to be able to share in the profits from the market for large aircraft, hitherto dominated by Boeing (at present with the 747-400), with the potential this has given Boeing to cross-subsidise its smaller aircraft.

For new aircraft the manufacturers have to compete in terms of operating cost and potential revenue, as well as performance, most obviously range and payload. The proposal to increase the size of an aircraft is not without special additional constraints on size; currently the 'box' allowed at major airports is 80 m × 80 m and this limits both the length and the wingspan. In addition there are strong incentives to avoid making the fuselage higher from the ground because of the consequences for ground handling. The aspect ratio of a wing (the ratio of span to chord) has a large effect on its drag and Airbus have until now had a larger aspect ratio than Boeing, a feature which has contributed to the lower drag of Airbus aircraft. With the A380 the limit on wing span to fit in the airport 'box' has meant that its aspect ratio of 7.53 will be lower than that of the 747-400, which is 7.98. It is still reasonable to expect that the cruise lift/drag ratio for the A380 will be around 20, significantly higher than the much older 747-400.

It is essential to realise that both the new aircraft, and the engines which power it, will depend heavily on the experience gained in earlier products, particularly those of similar size and character. Most of the aircraft that Airbus have made to date have two engines (referred to as twins) and only their A340 has four engines. Airbus will be relying on their knowledge and experience gained with the earlier aircraft, but most significantly the A340-600, which had its first flight in April 2001. This is certainly a large aircraft, with a maximum take-off weight of 365 tonne, not far short of the 747-400 with a maximum take-off weight of 395 tonne. Airbus will also be looking to learn from the 747-400. In Table 1.2 below the proposed specifications for the hypothetical New Large Aircraft on which the first part of this book is based are set beside those achieved for the 747-400 as well as for the A340-500 and A340-600. For the new aircraft some of the quantities given are stipulations, such as the number of passengers and the range, whilst others, such as the lift/drag ratio (discussed below) are extrapolations of earlier experience. The proposed range of 8000 nautical miles makes possible non-stop flights between
cities throughout North America and most of the major Pacific-Rim cities, even when strong head winds are liable to be encountered.

### Table 1.2 Comparison of some salient aircraft parameters

<table>
<thead>
<tr>
<th></th>
<th>New Large Aircraft</th>
<th>Boeing 747-400</th>
<th>-500</th>
<th>-600</th>
</tr>
</thead>
<tbody>
<tr>
<td>No of passengers</td>
<td>620</td>
<td>400</td>
<td>313</td>
<td>380</td>
</tr>
<tr>
<td>Range (nautical miles)</td>
<td>8000</td>
<td>7300</td>
<td>8550</td>
<td>7500</td>
</tr>
<tr>
<td>Payload at this range (a) (tonne)</td>
<td>58.8</td>
<td>38.5</td>
<td>29.7</td>
<td>36.1</td>
</tr>
<tr>
<td>Max. take-off weight (d) (tonne)</td>
<td>635.6</td>
<td>395.0</td>
<td>365</td>
<td>365</td>
</tr>
<tr>
<td>Empty weight* (b) (tonne)</td>
<td>298.7</td>
<td>185.7</td>
<td>170</td>
<td>177</td>
</tr>
<tr>
<td>Max. weight of fuel (c) (tonne)</td>
<td>275.4</td>
<td>174.4</td>
<td>171</td>
<td>157</td>
</tr>
<tr>
<td>Cruise Mach number</td>
<td>0.85</td>
<td>0.85</td>
<td>0.83</td>
<td>0.83</td>
</tr>
<tr>
<td>Initial cruise altitude (feet)</td>
<td>31000</td>
<td>31000</td>
<td>31000</td>
<td>31000</td>
</tr>
<tr>
<td>Cruise Lift/Drag</td>
<td>20</td>
<td>17.5</td>
<td>19.5</td>
<td>19.5</td>
</tr>
<tr>
<td>Wing area (m²)</td>
<td>790</td>
<td>511</td>
<td>439</td>
<td>439</td>
</tr>
</tbody>
</table>

(Note that $d = a + b + c$)

* no fuel, no payload

1 tonne = 2205 lb mass

In calculating payload one passenger is taken to be 95 kg, a similar value in pounds is specified by Boeing. In looking at the specifications for the new aircraft it is worth noting that the maximum payload is only 58.8 tonne, compared to the total weight of the new aircraft at take off, 635.6 tonne. More seriously, the payload (the total weight of passengers and freight) is not much more than one third of the fuel load. It follows from this that small proportional changes in the weight of the engine (which is 5 - 6% of the maximum take-off weight) or in the fuel consumption can have disproportionately large effects on the payload. Of the fuel carried not all could be used in a normal flight; typically about 15% ($\approx 38.6$ tonne) would need to be kept as reserve in case landing at the selected destination airport is impossible. Based on past experience it may be assumed that about 4% (around 11 tonne) of fuel would be used in take off and climb to the initial cruising altitude, with the bulk of the fuel consumption being involved in the cruise portion of a long flight.

### 1.3 Propulsion for the New Large Aircraft

It takes several years to design, develop, and certificate (i.e. test so that the aircraft is approved as safe to enter service) a new aircraft, though the length of time is becoming shorter. It seems to take even longer to develop the engines, but until the specifications of the aircraft are settled it is not clear what engine is needed. There are three major engine manufacturers (Rolls-Royce in
Britain, Pratt & Whitney and General Electric in the USA) and it is their aim to have an engine ready for whatever new large aircraft it is decided to build. The costs of developing a wholly new engine are so high that it is always the objective of a manufacturer to use whenever possible an existing engine, perhaps with some uprating. On a recent new large aircraft, the Boeing 777, all three major manufacturers offered an engine and the competition was fierce. Pratt & Whitney and Rolls-Royce offered developments of existing large engines; General Electric developed a wholly new engine, the GE90. The Economist of 18 September 1999 reported that the GE90 had cost General Electric $1 million per day for 4½ years, in total about $1.6 billion; it is not clear how much extra was spent by risk sharing partner companies. This huge sum can be made more understandable if an average wage for an employee, with the appropriate overheads, is taken to be $150,000 per annum - the $1.6 billion cost then translates into over 10,000 man-years of work. To reduce the financial exposure Pratt & Whitney and General Electric have formed an alliance to produce a wholly new engine for the A380, the GP7200, in competition with Rolls-Royce, who have offered the Trent 900, a derivative of their earlier engines.

Whilst discussions are going on between aircraft manufacturers and airlines they are also going on between aircraft manufacturers and the engine manufacturers. As specifications for the 'paper' aircrafts alter, the 'paper' engines designed to power them will also change; many potential engines will be tried to meet a large number of proposals for the new aircraft before any company finally commits itself. The first ten chapters of the book will attempt, in a very superficial way, to take a specification for an aircraft and design the engines to propel it – this is analogous, in a simplified way, to what would happen inside an engine company.

Because engines are large and heavy there are good aerodynamic and structural reasons for mounting engines under the wing. For example, a Rolls-Royce Trent 800, which is the lightest engine to power the Boeing 777, weighs about 8.2 tonne when installed on the aircraft. Most of the lift is generated by the wings, so hanging the comparatively massive engines where they can most easily be carried makes good structural sense. This reduces the wing root bending moment and makes possible a reduction in the strength and weight of the whole aircraft. It is the trend for new engines to be bigger and heavier for the same thrust than the ones they replace, originally to reduce fuel consumption, but now mainly to reduce noise. This will be discussed later in Chapter 7 and in the Appendix.

The A380 is to have four engines, two slung under each wing and the same arrangement is adopted here for the New Large Aircraft. Not very long ago it would have been unthinkable to have a trans-oceanic aircraft with only two engines because the reliability of the engines was inadequate. Now two-engine aircraft are very common, being the dominant type now crossing the Atlantic, but four engines offer advantages for the New Large Aircraft for two reasons. First, every aircraft must be able to climb from take off with one engine totally disabled. For a two-engine aircraft this means that there must be twice as much thrust available at take off as that just necessary to get the aircraft safely into the air. The engines must therefore be oversized for take off, implying too much available thrust at cruise (and therefore excess weight) with the
Part 1: Design of Engines for a New 600-seat Aircraft

engines 'throttled back'. For a four-engine aircraft the same rule requires that there is only \(4/3\) times as much thrust available at take off, and for aircraft designed for very long flights it is desirable to carry as little surplus weight as possible. The success of the Boeing 777 as a very long range aircraft has undermined this argument in recent years; as is discussed in later chapters the apparent disadvantage with two engines can be mitigated by cruising at higher altitude and the benefits in reduced first cost and maintenance cost compensates for a small increase in fuel consumption.

The second reason for having four engines is that it is not considered practical to make the wing much higher off the ground than current aircraft like the 747-400, since to do so would raise the cabin and if the cabin were raised higher the existing passenger handling facilities at airports would be unusable; it would also make the undercarriage much bigger and heavier. If the New Large Aircraft, or the Airbus A380, were to have only two engines these would be too large to fit under the wings at their current height from the ground.

It should be added in parenthesis, however, that because engines are expensive to buy and to maintain it is likely that smaller aircraft than the New Large Aircraft we are considering here will have only two engines, even when they are to be operated over large distances. Recent examples are the Airbus 330 and the Boeing 777; both of these large twins are used for flights that are sufficiently long that until recently a four-engine aircraft would have been needed.

1.4 THE UNITS USED

In Table 1.1 a number of the quantities are in non-SI units. This is common because the industry is dominated by the United States which has been rather slow to see the advantages of SI units. It is helpful to remember that

\[
\begin{align*}
1 \text{ lb mass} &= 0.4536 \text{ kg}, \quad 1000 \text{ kg} = 1 \text{ tonne} \\
1 \text{ lb force} &= 4.448 \text{ N} \\
1 \text{ foot} &= 0.3048 \text{ m} \quad \text{(altitude in feet used for air traffic control)} \\
1 \text{ nautical mile} &= 1.829 \text{ km} \quad \text{(nautical mile abbreviated to nm)} \\
1 \text{ knot} &= 1 \text{ nm/hour} = 0.508 \text{ m/s}
\end{align*}
\]

The nautical mile (abbreviated to nm) is not arbitrary in the way other units are, but is the distance around the surface of the earth corresponding to 1 minute of latitude (North–South). Treating the earth as a sphere this is equivalent to 1 minute of longitude (East–West) around the equator. (The circumference of the earth around the equator, or any other great circle, is therefore \(360 \times 60\) nautical miles.)

The data in Table 1.1 also give the cruising speed as a Mach number, defined as \(V/a\) the ratio of the flight speed \(V\) to the local speed of sound \(a\). Wherever possible aerodynamicists use
non-dimensional numbers and Mach number is one of the most important in determining the performance of the aircraft. The speed of sound is given by

\[ a = \sqrt{\gamma R T} \]

where \( T \) is the local atmospheric temperature (i.e. the static temperature) \( \gamma \) is the ratio of the specific heats \( c_p/c_v \) (which is taken here to be 1.40 for air) and \( R \) is the gas constant (0.287 kJ/kg K for air). Since \( c_p = \gamma R/\gamma - 1 \) this leads to \( c_p = 1.005 \) kJ/kg. These values will be used for the atmosphere and in Part 1 (Chapters 1–10) for the gas in the engine. These values would not be accurate enough for use in a real design, particularly for the products of combustion, but also for pure air at elevated temperatures. Although this simplification suffices for the treatment in Part 1 of the book it will be relaxed in later parts.

**Exercise**

1.1 The shortest distance between two places on the surface of the earth is the *Great Circle Distance*, which, for a perfectly spherical earth, would be equal to the radius \( R_e \) of the earth times the angle \( A \) subtended between vectors from the centre of the earth to the points on the surface.

Express the positions of points 1 and 2 on the surface of the earth in terms of Cartesian vectors about the centre of the earth, using \( \theta_1 \) and \( \phi_1 \) to denote the latitude and longitude respectively for point 1 and likewise \( \theta_2 \) and \( \phi_2 \) for point 2. Then take the dot product of the vectors to show that the cosine of the angle \( A \) is given by \( \cos A = \cos \theta_1 \cos \theta_2 \cos (\phi_1 - \phi_2) + \sin \theta_1 \sin \theta_2 \).

Find the shortest distance in nautical miles between London (latitude 51.5° N, longitude 0) and Sydney in Australia (latitude 33.9° South, longitude 151.3° East). *(Ans: 9168 nm)*

**1.5 The Standard Atmosphere**

The atmosphere through which the aircraft flies depends on the altitude, with the pressure, temperature and density falling as altitude increases. The temperature profile with height is determined primarily by the absorption of solar radiation by water vapour and subsequent radiation back into space. At high altitude the variation with season, location and time of day is much less than at ground level and it is normal to use a standard atmosphere in considering aircraft and engine performance. Temperature, density and pressure are plotted in Fig.1.1 according to the *International Standard Atmosphere* (ISA). Standard sea-level atmospheric conditions are defined as \( T_{sl} = 288.15 \) K, \( p_{sl} = 101.3 \) kPa, \( \rho_{sl} = 1.225 \) kg/m³. In the standard atmosphere temperature is assumed to decrease linearly with altitude at 6.5 K per 1000 m below the *tropopause* (which in the standard atmosphere is assumed to be at 11000 m, that is 36089 feet), but to remain constant above this altitude at 216.65 K. (The discontinuity in temperature gradient must give a discontinuity in the pressure and density gradients too, but this is small and the curve fitting programme has smoothed it out.)
As noted above, non-SI units are common in aviation, and air traffic control assigns aircraft to corridors at altitudes defined in feet. Cruise very often begins at 31000 ft, and the corridors are separated by 2000 ft. Although this book will be based on SI units, altitudes for the civil aircraft will be given in feet. Table 1.3 may be helpful.

Table 1.3 Useful values of the International Standard Atmosphere*

<table>
<thead>
<tr>
<th>Altitude feet</th>
<th>Temperature K</th>
<th>Pressure $10^5$ Pa</th>
<th>Density kg/m$^3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>288.15</td>
<td>1.013</td>
</tr>
<tr>
<td>31000</td>
<td>9.45</td>
<td>226.73</td>
<td>0.287</td>
</tr>
<tr>
<td>33000</td>
<td>10.05</td>
<td>222.82</td>
<td>0.260</td>
</tr>
<tr>
<td>35000</td>
<td>10.67</td>
<td>218.80</td>
<td>0.238</td>
</tr>
<tr>
<td>37000</td>
<td>11.28</td>
<td>216.65</td>
<td>0.214</td>
</tr>
<tr>
<td>39000</td>
<td>11.88</td>
<td>216.65</td>
<td>0.197</td>
</tr>
<tr>
<td>41000</td>
<td>12.50</td>
<td>216.65</td>
<td>0.179</td>
</tr>
<tr>
<td>51000</td>
<td>15.54</td>
<td>216.65</td>
<td>0.110</td>
</tr>
</tbody>
</table>

*Also known as the ICAO Standard Atmosphere.

Figure 1.1 The International Standard Atmosphere
The new large aircraft – requirements and background

For the purpose of this book the conditions of the standard atmosphere will be assumed to apply exactly – this makes for consistency in the numbers and facilitates checking the exercises. It will be clear, however, that the standard atmosphere is at best an approximation to conditions averaged over location and season. The temperature varies more than the pressure and this variation is greatest close to the ground. It is not uncommon, for example, for the temperature at an airport in continental North America to be as low as −40 °C in winter and as high as +40 °C in summer. It is normal to refer to conditions relative to the standard atmosphere, so that if at 31000 feet altitude the temperature were 236.7 K it could, by reference to Table 1.3, be described as ISA+10°C. The corrections from standard conditions are often large for high altitude airports. Johannesburg airport, for example, is 5557 feet above sea level and the ISA temperature for this altitude is 4.0°C: suppose on a hot day that the temperature at Johannesburg airport were 35°C – in this case the conditions would be described as ISA+31°C.

Exercises

1.2 Express the maximum take off weight (mtow) for the New Large Aircraft in pounds (the units that much of the airline industry uses). Make a rough estimate of the flight time for a range of 8000 nm if cruise were at the initial altitude and Mach number for the whole flight.

\( \text{(Ans: range = 14632 km; altitude = 9448 m; mtow = 1.4 \cdot 10^6 \text{ lb}; time of flight = 15.8 \text{ hours})} \)

1.3 Find the cruising speed in m/s and km/h corresponding to the specified cruise Mach number and the initial cruise altitude. If the altitude at the end of the flight is 41000 ft, \((p_a = 17.9 \text{ kPa}, T_a = 216.7 \text{ K})\) what is the flight speed then for the same Mach number. (Note air traffic control usually allots aircraft cruising altitudes in 2000 ft steps: 31000, 35000 and 39000 going from East to West, and 33000, 37000 and 41000 going from West to East.)

\( \text{(Ans: Initial speed at 31000 ft, 256.5 m/s, 923 km/h; at 41000 feet, speed 250.8 m/s, 903 km/h.)} \)

1.4 The pressure change with altitude \( h \) due to hydrostatic effects is given by \( dp = -\rho g dh \).

\( a) \) For an idealised atmosphere the temperature falls with altitude at a constant rate so that \( \frac{\partial T}{\partial h} = -k \), where \( k \) is a constant with units K/m. Show that the pressure \( p \) at altitude \( H \) can be written

\[ p = p_{sl}\left(1-kH/T_{sl}\right)^{g/Rk} = p_{sl}\left(T/T_{sl}\right)^{g/Rk} \]

where \( p_{sl} \) and \( T_{sl} \) are the static pressure and temperature at sea level, 101.3 kPa and 288.15 K.

For the International Standard Atmosphere the rate of change in temperature with altitude is taken to be 6.5 K per 1000 m up to the tropopause at 11 km. Show that when \( g = 9.81 \text{ m/s}^2 \) and \( R = 287 \text{ J/kgK} \), the pressure at altitude \( H \), in metres, is given by

\[ p = p_{sl}\left(T/T_{sl}\right)^{5.26} = p_{sl}\left(1-2.26 \times 10^{-5}H\right)^{5.26} \]

up to the tropopause, above which the pressure is given by

\[ p = p_T \exp\{-1.58 \times 10^{-4}(H-11.10^3)\} \]

where \( p_T \) is the pressure at the tropopause.

\( b) \) If the relationship between pressure and density were that for isentropic changes (i.e. reversible and adiabatic) \( p/\rho^\gamma = \text{constant} \), show that the pressure at altitude \( H \) can then be written as

\[ p = p_{sl}\left[1 - \frac{\gamma-1}{\gamma} \frac{gH}{RT_{sl}}\right]^\left(\frac{\gamma}{\gamma-1}\right) \]

* Exercises with an asterix produce solutions which should, for convenience, be entered on the Design Sheet at the back of the book.

Note that to maintain consistency and to make checking of solutions easier, answers are given to a precision which is much greater than the accuracy of the assumptions warrants.
Plot a few values of pressure, density and temperature on Fig.1.1, the International Standard Atmosphere.

Notes: Atmospheric air is not dry. For saturated air the rate of temperature drop is given as 4.9 K per km, compared with 6.5 K per km in the International Standard Atmosphere. The isentropic calculation assumed dry air.

Different 'standard' atmospheres are sometimes used to model situations more closely: for example over Bombay in the monsoon season the atmosphere is very different from over Saudi Arabia in summer or northern Russia or America in winter.

Even below the tropopause the standard atmosphere assumes a slower reduction in temperature with altitude than that which would follow from an isentropic relation between pressure and temperature; the standard atmosphere is therefore stable. To understand this, imagine the atmosphere perturbed so that a packet of air is made to rise slowly. As the packet rises its pressure will fall to be equal to the pressure of the air that surrounds it and, as a reasonable approximation, the temperature and pressure for the packet of air will be related by the isentropic relation \( p/T^\gamma/(\gamma-1) = \text{constant} \). If the ascending air, which has an isentropic relation between temperature and pressure, were slightly warmer than its surroundings it would be less dense than the surrounding air and would continue to rise; such an atmosphere would be unstable. If, on the other hand, the ascending packet of air has a temperature lower than that of its immediate surroundings, as occurs in the standard atmosphere, it would be denser than the surrounding air and would fall back; such an atmosphere would be stable. In the first few hundred metres above the ground the convection frequently tends to make the atmosphere locally unstable, which is useful because it helps disperse pollutants. Stable atmospheres can occur near ground level, and frequently do at night under windless conditions when radiation leads to the ground cooling more rapidly than the air above it. Under stable conditions near the ground the natural mixing of the atmosphere is suppressed and the conditions for fog and pollution build-up are liable to occur.

1.5 ENVIRONMENTAL ISSUES

When jet propelled passenger transport was initiated, little or no thought was given to the environment, either near the airports or in the upper atmosphere. By the late 1960s the situation near airports was becoming intolerable, mainly because of the noise, but also because of pollution. The pollution involved unburned hydrocarbons, smoke (i.e. small particles of soot, which is unburned carbon) and oxides of nitrogen. Gradually steps have been taken to rein in these nuisances by international agreement with regulations both for combustion product emissions near airports and for noise during take off and landing.

The international agreements are reached so that the interests of various parts of the industry (from manufacturers of engines through to the airlines which operate rather old aircraft) are addressed. The net result is that the international agreements have lagged behind public
pressure for amelioration and as a result local regulations at important airports around the world have tended to be more challenging for the makers of new engines to meet. The international limits on noise are so far above the noise produced by new aircraft with modern engines that the international limit serves merely as the benchmark from which the margin of lower noise is set. For noise the airport which tends to determine the level which new large aircraft have to achieve is London Heathrow. For products of combustion an airport which sets the level is Zurich, where charges are varied depending on the amount of pollution released in a standard landing and take-off operation. The issues and rules for emissions of pollutants are addressed briefly in Section 11.5. Noise is considered in an appendix at the end of the book.

The effect of regulations for combustion emissions has not, so far, had very much effect on the overall layout of the engine. General Electric have used a staged combustor (like two connected annular combustion chambers, one used all the time and the other only for high power) on their GE90 and it is available on the CFM-56, but so far other manufacturers have managed to avoid even this change by attention to detail in a more conventional single combustor. The 1999 report by the Intergovernmental Panel on Climate Change (IPCC)\(^2\) may lead to greater pressure for control of oxides of nitrogen, amongst other things, during the cruise. The effect of noise regulation, however, has recently lead to very significant alterations to the engine, with consequent reduction in aircraft performance and a slightly larger fuel burn. Principally this is because at take off the largest noise source is still produced by the jet, and no method of jet-noise reduction is more certain than reducing the jet velocity. This requires bigger engines for the same thrust, engines which are bigger than those which would be chosen for optimum aircraft range, a topic taken further in Chapter 7.

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**SUMMARY CHAPTER 1**

New engines are extremely expensive to develop and the risk of designing an engine for an aircraft which does not get built is a serious concern. Unfortunately the time to design and develop the engines has in the past been greater than for the airframe. The hypothetical New Large Aircraft which forms the basis of the first 10 chapters of this book bears some resemblance to the Airbus A380 currently under design. For such a large aircraft, intended for very long range operation, there will be four engines slung under the wings.

There is an International Standard Atmosphere used for calculating aircraft performance, which gives temperature, pressure and density as a function of altitude. Temperature is assumed to fall linearly with altitude (at 6.5 K per km) until 11 km, beyond which it is constant to 20 km.

\(^2\)See bibliography
Subsonic civil air transport does not normally fly above 41000 ft (12.5 km), though business jets fly at up to 51000 ft (15.5 km). The atmospheric temperature normally falls more slowly with altitude than is implied by an isentropic variation between temperature and pressure.

Whenever possible non-dimensional variables are used, such as Mach number. When non-dimensional variables cannot be used SI units will be used throughout the book unless there is a clear reason otherwise (e.g. feet for altitude and nautical miles for range).

Environmental issues are becoming more important, with the emphasis in regulations currently being around the airport. The potentially more serious effects of emissions in the upper atmosphere will probably be the subject of future regulation. Limiting noise during take off and landing has already lead to the engine layout being modified so that it is no longer optimum for range or fuel consumption.

Chapter 1 sets out to define the needs, the operating environment and the broad specification of the aircraft. Chapter 2 moves to the next stage, which is to consider the aircraft itself.