Gravity
from the ground up

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Gravity on Earth: the inescapable force

Gravity is everywhere. No matter where you go, you can’t seem to escape it. Pick up a stone and feel its weight. Then carry it inside a building and feel its weight again: there won’t be any difference. Take the stone into a car and speed along at 100 miles per hour on a smooth road: again there won’t be any noticeable change in the stone’s weight. Take the stone into the gondola of a hot-air balloon that is hovering above the Earth. The balloon may be lighter than air, but the stone weighs just as much as before.

This inescapability of gravity makes it different from all other forces of nature. Try taking a portable radio into a metal enclosure, like a car, and see what happens to its ability to pick up radio stations: it gets seriously worse. Radio waves are one aspect of the electromagnetic force, which in other guises gives us static electricity and magnetic fields. This force does not penetrate everywhere. It can be excluded from regions if we choose the right material for the walls. Not so for gravity. We could build a room with walls as thick as an Egyptian pyramid and made of any exotic material we choose, and yet the Earth’s gravity would be right there inside, as strong as ever. Gravity acts on everything the same way.

Every body falls toward the ground, regardless of its composition. We know of no substance that accelerates upwards because of the Earth’s gravity. Again this distinguishes gravity from all the other fundamental forces of Nature. Electric charges come in two different signs, the “+” and “−” signs on a battery. A negative electron attracts a positive proton but repels other electrons.

The existence of two signs of electric charge is responsible for the shape of our everyday world. For example, the balance between attraction and repulsion among the different charges that make up, say, a piece of wood gives it rigidity: try to stretch it and the electrons resist being pulled away from the protons; try to compress it and the electrons resist being squashed up against other electrons. Gravity allows no such fine balances, and we shall see that this means that bodies in which gravity plays a dominant role cannot be rigid. Instead of achieving equilibrium, they have a strong tendency to collapse, sometimes even to black holes.

These two facts about gravity, that it is ever-present and always attractive, might make it easy to take it for granted. It seems to be just part of the background, a constant and rather boring feature of our world. But nothing could be further from the truth. Precisely because it penetrates everywhere and cannot be cancelled out, it
Chapter 1. Gravity on Earth

is the engine of the Universe. All the unexpected and exciting discoveries of modern astronomy – quasars, pulsars, neutron stars, black holes – owe their existence to gravity. It binds together the gases of a star, the stars of a galaxy, and even galaxies into galaxy clusters. It has governed the formation of stars and it regulates the way stars create chemical elements of which we are made. On a grand scale, it controls the expansion of the Universe. Nearer to home, it holds planets in orbit about the Sun and satellites about the Earth.

The study of gravity, therefore, is in a very real sense the study of practically everything from the surface of the Earth out to the edge of the Universe. But it is even more: it is the study of our own history and evolution right back to the Big Bang. Because gravity is everywhere, our study of gravity in this book will take us everywhere, as far away in distance and as far back in time as we have scientific evidence to guide us.

Galileo: the beginnings of the science of gravity

We will begin our study of gravity with our feet firmly on the ground, by meeting a man who might fairly be called the founder of modern science: Galileo Galilei (1564–1642).

In Galileo’s time there was a strong interest in the trajectories of cannonballs. It was, after all, a matter of life and death: an army that could judge how far gravity would allow a cannonball to fly would be better equipped to win a battle over a less well-informed enemy. Galileo’s studies of the trajectory problem went far beyond those of any previous investigator. He made observations in the field and then performed careful experiments in the laboratory. These experiments are a model of care and attention to detail. He found out two things that startled many people in his day and that remain cornerstones of the science of gravity.

First, Galileo found that the rate at which a body falls does not depend upon its weight. Second, he measured the rate at which bodies fall and found that their acceleration is constant, independent of time.

After Galileo, gravity suddenly wasn’t boring any more. Let’s look at these two discoveries to find out why.

The story goes that Galileo took two iron balls, one much heavier than the other, to the top of the bell tower of Pisa and dropped them simultaneously. Most people of the day (and even many people today!) would probably have expected the heavier ball to have fallen much faster than the lighter one, but no: both balls reached the ground together.

The equality of the two balls’ rates of fall went against the intuition and much of the common experience of the day. Doesn’t a brick fall faster than a feather? Galileo pointed out that air resistance can’t be neglected in the fall of a feather, and that to discover the properties of gravity alone we must experiment with dense bodies like stones or cannonballs, where the effects of air resistance are small. For such objects we find that speed is independent of weight.

But surely, one might object, we have to do much more work to lift a heavy stone than a light one, so doesn’t this mean that a heavy stone “wants” to fall more than a light one and will do so faster, given the chance? No, said Galileo: weight has nothing to do with the speed of fall. We can prove that by measuring it. We have to accept the world the way we find it. This was the first step towards what we now call the principle of equivalence, which essentially asserts that gravity is indistinguishable from uniform acceleration. We shall see that this principle has a remarkable number of consequences, from the weightlessness of astronauts to the possibility of black holes.
The acceleration of gravity is uniform

Investigation 1.1. Faster and faster: the meaning of uniform acceleration

In this investigation, we work out what Galileo’s law of constant acceleration means for the speed of a falling body. The calculation is short, and it introduces us to the way we will use some mathematical symbols through the rest of the book.

We shall denote time by the letter \( t \) and the speed of the falling body by \( v \) (for velocity). The speed at time \( t \) will be written \( v(t) \). The acceleration of the body is \( g \), and it is constant in time.

Suppose the body is dropped from rest at time \( t = 0 \). Then its initial speed is \( v = 0 \) at time \( t = 0 \), in other words \( v(0) = 0 \). What will be its speed a short time later?

Let us call this later time \( \Delta t \). Here we meet an important new notation: the symbol \( \Delta \) will always mean “a change in” whatever symbol follows it. Thus, a change in time is \( \Delta t \). Similarly, we shall call the change in speed produced by gravity \( \Delta v \). Normally we shall use this notation to denote small changes; here, for example, I have defined \( \Delta t \) to be “a short time later”. We shall ask below how small \( \Delta t \) has to be in order to be “short”.

The acceleration \( g \) is the change in the speed per unit time. This definition can be written algebraically as

\[
g = \frac{\Delta v}{\Delta t} \quad (1.1)
\]

By multiplying through by the denominator of the fraction, we can solve for the change in speed:

\[
\Delta v = g \Delta t. \quad (1.2)
\]

Exercise 1.1.1: Speed of a falling body

Using the fact that the acceleration of gravity on Earth is \( g = 9.8 \text{ m s}^{-2} \), calculate the speed a ball would have after falling for two seconds, if dropped from rest. Calculate its speed if it were thrown downwards with an initial speed of \( 10 \text{ m s}^{-1} \). Is it falling or still rising after \( 2 \text{ s} \)?

The acceleration of gravity is uniform

Galileo performed a number of ingenious experiments with the rather crude clocks available in his day to demonstrate that the acceleration of falling objects is constant. Now, the acceleration of an object is the rate of change of its speed, so if the acceleration is constant then the speed changes at a constant rate; during any given single second of time, the speed increases by a fixed amount. We call this constant the acceleration of gravity, and denote it by \( g \) (for gravity). Its value is roughly 9.8 meters per second per second. The units, meters per second per second, should be understood as “(meters per second) per second”, giving the amount of speed (meters per second) picked up per second. These units may be abbreviated as m/s/s, but it is more conventional and avoids the ambiguous ordering of division signs to write them as m s\(^{-2}\).

As with any physical law, there is no reason “why” the world had to be this way: the experiment might have shown that the speed increased uniformly with the distance fallen. But that is not how our world is made. What Galileo found was that speed increased uniformly with time of fall.

We can find out what Galileo’s law says about the distance fallen by doing our first calculations, Investigation 1.1 and 1.2. These calculations show that uniform acceleration implies that the speed a falling body gains is proportional to time and that the distance it falls increases as the square of the time. The calculation also has another purpose: it introduces the basic ideas and notation that we will use in later investigations to construct computer calculations of more complicated phe-

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1Ambiguity: does m/s/s mean (m/s)/s or m/(s/s)? Either would be a valid interpretation of m/s/s, but in the second form the units for seconds cancel, which is not at all what is wanted.
In this section: Galileo introduced the idea that the horizontal and vertical motions of a body can be treated separately: the vertical acceleration of gravity does not change the horizontal speed of a body.
Trajectories of cannonballs

Here we show how the finite-differences reasoning of the two previous investigations allows us to construct a computer program to calculate the flight of a cannonball, at least within the approximation that the ball is not affected by air resistance.

From this book’s website you can download listing of the Java program CannonTrajectory. If you download the Triana software as well you can run the program and compute the trajectory of a cannonball fired at any given initial speed and at any angle. Figure 1.3 on the next page displays the result of the computer calculation for three trajectories, all launched with the same speed at three different angles. (The Triana software will produce plots of these trajectories. The reader is encouraged to re-run the program with various initial values of V to check this result.

Exercise 1.3.1: Small steps in speed and distance

Suppose that at the n\textsuperscript{th} time-step t\textsubscript{n}, the vertical speed is v\textsubscript{y} and the vertical distance above the ground is h\textsubscript{n}. Show that at the next time-step t\textsubscript{n+1} = t\textsubscript{n} + ∆t, the vertical speed is

\[ v_{y_{n+1}} = v_{y_{n}} + g \Delta t. \]

Using our method of approximating the distance traveled by using the average speed over the interval, show that at the next time-step the height will be

\[ h_{n+1} = h_{n} + \frac{1}{2}(v_{y_{n}} + v_{y_{n+1}})\Delta t = h_{n} + v_{y_{n}}\Delta t - \frac{1}{2}g(\Delta t)^{2}. \]

Exercise 1.3.2: Suicide shot

What is the minimum range of a cannonball fired with a given speed V, and at what angle should it be aimed in order to achieve this minimum?

Exercise 1.3.3: Maximum range by algebra

For readers interested in verifying the guess we made above from the numerical data, here is how to calculate the range at 45\degree angle algebraically. The range is limited by the amount of time the cannonball stays in the air. Fired at 45\degree with speed V, how long does it take to reach its maximum height, which is where its vertical speed goes to zero? Then how long does it take to return to the ground? What is the total time in the air? How far does it go horizontally during this time? This is the maximum range.

Exercise 1.3.4: Best angle of fire

Prove that 45\degree is the firing angle that gives the longest range by calculating the range for any angle and then finding what angle makes it a maximum. Use the same method as in Exercise 1.3.3.
But instead we show in Investigation 1.3 on the previous page how to use a personal computer to calculate the actual trajectory of a cannonball. These computer techniques will form the foundation of computer programs later in this book that will calculate other trajectories, such as planets around the Sun, stars in collision with one another, and particles falling into black holes.

**Galileo: the first relativist**

It would be hard to overstate Galileo’s influence on science and therefore on the development of human society in general. He founded the science of mechanics; his experiments led the English scientist Isaac Newton (1642–1726) to discover his famous laws of motion, which provided the foundations for almost all of physics for 200 years. And almost 300 years after his death his influence was just as strong on Albert Einstein. The German–Swiss physicist Einstein (1879–1955) replaced Newton’s laws of motion and of gravity with new ones, based on his theory of relativity. Einstein’s revolutionary theories led to black holes, the Big Bang, and many other profound predictions that we will study in the course of this book. Yet Einstein, too, kept remarkably close to Galileo’s vision.

The main reason for Galileo’s influence on Einstein is that he gave us the first version of what we now call the principle of relativity. We have already encountered Galileo’s version: the vertical motion of a ball does not depend on its horizontal speed, and its horizontal speed will not change unless a horizontal force is applied. Where we used a fast-flying airplane to justify this, Galileo imagined a sailing ship on a smooth sea, but the conclusion was the same: an experimenter moving horizontally will measure the same acceleration of gravity in the vertical direction as he would if he were at rest.

Galileo took this idea and drew a much more profound conclusion from it. The radical proposal made half a century earlier by the Polish priest and astronomer Nicolas Copernicus (1473–1543), that the Earth and other planets actually moved around the Sun (see Figure 1.4), was still far from being accepted by most intellectuals in Galileo’s time. Although the proposal explained the apparent motions of the planets in a simple way, it was open to an important objection: if the Earth is moving at such a rapid rate, why don’t we feel it? Why isn’t the air left behind, why doesn’t a ball thrown vertically fall behind the moving Earth?

Galileo used the independence of different motions to dispose of this objection. Galileo’s answer is that a traveler in the cabin of a ship on a smooth sea also does not feel his ship’s motion: all the objects in the cabin move along with it at constant speed, even if they are just resting on a table and not tied down. Anything that falls will fall vertically in the cabin, giving no hint of the ship’s speed. So it is on the Earth, according to Galileo: the air, clouds, birds, trees, and all other objects all have the same speed, and this motion continues until something interferes with it. There is, in other words, no way to tell that the Earth is moving through space except to look at things far away, like the stars, and see that it is.
Today we re-phrase and enlarge this idea to say that all the laws of physics are just the same to an experimenter who moves with a uniform motion in a straight line as they are to one who remains at rest, and we call this the principle of relativity. We shall encounter many of its consequences as we explore more of the faces of gravity.

Unfortunately for Galileo, his clear reasoning and his observations with one of the first telescopes made him so dangerous to the established view of the Roman Catholic Church that in his old age he was punished for his views, and forced to deny them publicly. Privately he continued to believe that the planets went around the Sun, because he had discovered with his telescope that the moons of Jupiter orbit Jupiter in the same way that the planets orbit the Sun.

Today we recognize Galileo as the person who, more than anyone else, established the Copernican picture of the Solar System.

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**Figure 1.4.** The Copernican view of the planets known in Galileo’s day as they orbit the Sun.

**Figure 1.5.** Part of a sketch by Galileo of the positions of Jupiter (open circles) and its moons (stars) on a sequence of nights (dates given by the numbers). The big changes from night to night puzzled Galileo. At first he believed that Jupiter itself was moving erratically, but after a few observations he realized that the “stars” were moons orbiting Jupiter in the same way that the planets orbit the Sun.