

The Strange World of Quantum Mechanics

Daniel F. Styer
Oberlin College, Ohio



PUBLISHED BY THE PRESS SYNDICATE OF THE UNIVERSITY OF CAMBRIDGE
The Pitt Building, Trumpington Street, Cambridge, United Kingdom

CAMBRIDGE UNIVERSITY PRESS
The Edinburgh Building, Cambridge CB2 2RU, UK
40 West 20th Street, New York, NY 10011-4211, USA
477 Williamstown Road, Port Melbourne, VIC 3207, Australia
Ruiz de Alarcón 13, 28014 Madrid, Spain
Dock House, The Waterfront, Cape Town 8001, South Africa

<http://www.cambridge.org>

© Daniel F. Styer 2000

This book is in copyright. Subject to statutory exception
and to the provisions of relevant collective licensing agreements,
no reproduction of any part may take place without
the written permission of Cambridge University Press.

First published 2000
Reprinted 2000, 2003

Printed in the United Kingdom at the University Press, Cambridge

Typeface Times 11/13pt. *System* L^AT_EX [UPH]

A catalogue record for this book is available from the British Library

Library of Congress Cataloguing in Publication data

Styer, Daniel F.

The strange world of quantum mechanics / Daniel F. Styer.

p. cm.

Includes index.

ISBN 0 521 66104 8. – ISBN 0 521 66780 1 (pbk.)

1. Quantum theory. I. Title

QC174.12.S879 1999

530.12-dc21 99-13559 CIP

ISBN 0 521 66104 8 hardback

ISBN 0 521 66780 1 paperback

Contents

Preface	xi
1 Introduction	1
Classical mechanics describes how ordinary-sized things behave. Quantum mechanics describes how atomic-sized things behave.	
2 Classical Magnetic Needles	5
In classical mechanics, a compass needle behaves like a “magnetic arrow” that obeys certain rules.	
3 The Stern–Gerlach Experiment	13
Experiments show that atomic-sized magnetic needles do not behave exactly like arrows.	
4 The Conundrum of Projections; Repeated Measurements	21
In fact, atomic-sized magnetic needles can’t behave like arrows at all! Repeated measurement experiments suggest that only probabilities, not certainties, can be predicted in quantum mechanics.	
5 Probability	31
An understanding of probability is necessary for quantum mechanics and important for day-to-day life.	
6 The Einstein–Podolsky–Rosen Paradox	38
The probabilistic character of quantum mechanics, suggested previously, is here proved.	
7 Variations on a Theme by Einstein	49
Two more proofs, intellectual descendants of the Einstein–Podolsky–Rosen argument. (This chapter is optional.)	

8	Optical Interference	57
	Atomic-sized things don't behave in the familiar classical way. But how do they behave? Light provides a clue, in that light from two sources can add up to produce — not more light — but darkness.	
9	Quantal Interference	64
	We design an apparatus with two routes through which an atom may pass from the input to the output. If the atom must pass through one route, then the probability of passage is $\frac{1}{4}$. If it must pass through the other route, then the probability of passage is $\frac{1}{4}$. But if it may pass through either route, then the probability of passage is ... zero!	
10	Amplitudes	76
	Quantal interference is described using an abstract entity called “amplitude”.	
11	Working with Amplitudes	86
	Amplitude is represented mathematically by an arrow in a plane. Amplitude is associated with a process, not with a particle.	
12	Two-Slit Inventions	94
	Variations on the quantal interference experiment drive home the point that “the atom takes both routes”.	
13	Quantum Cryptography	98
	Quantum mechanics invites deep thought about the nature of reality and the character of science. But on the practical level, it also allows the construction of an unbreakable code. (This chapter is optional.)	
14	Quantum Mechanics of a Bouncing Ball	103
	The quantal rules for amplitudes, when applied to an ordinary-sized ball moving through space, give the same common-sense result as classical mechanics — unless we trick the ball!	
15	The Wavefunction	113
	How does an atom behave when it has no position? How can humans visualize this behavior?	
	Appendix A: A Brief History of Quantum Mechanics	119
	Appendix B: Putting Weirdness to Work	133
	Appendix C: Sources	138
	Appendix D: General Questions	141
	Appendix E: Bibliography	145
	Appendix F: Skeleton Answers for Selected Problems	149
	Index	151

1

Introduction

1.1 Capsule history of quantum mechanics

Starting in the seventeenth century, and continuing to the present day, physicists developed a body of ideas that describe much about the world around us: the motion of a cannonball, the orbit of a planet, the working of an engine, the crack of a baseball bat. This body of ideas is called *classical mechanics*.

In 1905, Albert Einstein realized that these ideas didn't apply to objects moving at high speeds (that is, at speeds near the speed of light) and he developed an alternative body of ideas called *relativistic mechanics*. Classical mechanics is wrong in principle, but it is a good approximation to relativistic mechanics when applied to objects moving at low speeds.

At about the same time, several experiments led physicists to realize that the classical ideas also didn't apply to very small objects, such as atoms. Over the period 1900–1927 a number of physicists (Planck, Bohr, Einstein, Heisenberg, de Broglie, Schrödinger, and others) developed an alternative *quantum mechanics*. Classical mechanics is wrong in principle, but it is a good approximation to quantum mechanics when applied to large objects.

1.2 What is the nature of quantum mechanics?

I'm not going to spend any time on the history of quantum mechanics, which is convoluted and fascinating. Instead, I will focus on the ideas developed at the end. What sort of ideas required twenty-eight years of development from this stellar group of scientists?

Einstein's theory of relativity is often (and correctly) described as strange and counterintuitive. Yet, according to a widely used graduate level text,

[the theory of relativity] is a modification of the structure of mechanics which must not be confused with the far more violent recasting required by quantum theory.

Murray Gell-Mann, probably the most prominent living practitioner of the field, said of quantum mechanics that

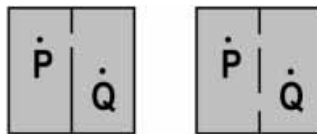
Nobody feels perfectly comfortable with it.

And the inimitable Richard Feynman, who developed many of the ideas that will be expounded in this book, remarked that

I can safely say that nobody understands quantum mechanics.

One strange aspect of quantum mechanics concerns predictability. Classical mechanics is *deterministic* — that is, if you know exactly the situation as it is now, then you can predict exactly what it will be at any moment in the future. Chance plays no role in classical mechanics. Of course, it might happen that the prediction is very difficult to perform, or it might happen that it is very difficult to find exactly the current situation, so such a prediction might not be a practical possibility. (This is the case when you flip a coin.) But in principle any such barriers can be surmounted by sufficient work and care. Relativistic mechanics is also deterministic. In contrast, quantum mechanics is *probabilistic* — that is, even in the presence of exact knowledge of the current situation, it is impossible to predict its future exactly, regardless of how much work and care one invests in such a prediction.

Even stranger, however, is quantum mechanical *interference*. I cannot describe this phenomenon in a single paragraph — that is a major job of this entire book — but I can give an example. Suppose a box is divided in half by a barrier with a hole drilled through it, and suppose an atom moves from point P in one half of the box to point Q in the other half. Now suppose a second hole is drilled through the barrier and then the experiment is repeated. The second hole increases the number of possible ways to move from P to Q, so it is natural to guess that its presence will increase the probability of making this move. But in fact — and in accord with the predictions of quantum mechanics — a second hole drilled at certain locations will *decrease* that probability.



The fact that quantum mechanics is strange does not mean that quantum mechanics is unsuccessful. On the contrary, quantum mechanics is the most

successful theory that humanity has ever developed; the brightest jewel in our intellectual crown. Quantum mechanics underlies our understanding of atoms, molecules, solids, and nuclei. It is vital for explaining aspects of stellar evolution, chemical reactions, and the interaction of light with matter. It underlies the operation of lasers, transistors, magnets, and superconductors. I could cite reams of evidence backing up these assertions, but I will content myself by describing a single measurement. One electron will be stripped away from a helium atom that is exposed to ultraviolet light below a certain wavelength. This threshold wavelength can be determined experimentally to very high accuracy: it is $50.425\,929\,9 \pm 0.000\,000\,4$ nanometers. The threshold wavelength can also be calculated from quantum mechanics: this prediction is $50.425\,931\,0 \pm 0.000\,002\,0$ nanometers. The agreement between observation and quantum mechanics is extraordinary. If you were to predict the distance from New York to Los Angeles with this accuracy, your prediction would be correct to within the width of your hand. In contrast, classical mechanics predicts that *any* wavelength of light will strip away an electron, that is, that there will be no threshold at all.

1.3 How small is small?

I said above that the results predicted by quantum mechanics differed from the results predicted by classical mechanics only when these ideas were applied to “very small objects, such as atoms”. How small is an atom? Cells are small: a typical adult contains about 60 trillion cells. But atoms are far smaller: a typical cell contains about 120 trillion atoms. An atom is twice as small, relative to a cell, as a cell is small, relative to a person. In this book, when I say “small” I mean “*very* small”. You’ve never handled objects this small; I’ve never handled objects this small; none of your friends has ever handled objects this small. They are completely outside the domain of our common experience. As you read this book, you will find that quantum mechanics is contrary to common sense. There is nothing wrong with this. Common sense applies to commonly encountered situations, and we do not commonly encounter the atomic world.

1.4 The role of mathematics in quantum mechanics

One frequently hears statements to the effect that the ideas of quantum mechanics are highly mathematical and can only be understood through the use of complex mathematics (partial differential equations, Fourier transforms, eigenfunction expansions, etc.).

One can popularize the quantum theory [only] at the price of gross oversimplification and distortion, ending up with an uneasy compromise between what the facts dictate and what it is possible to convey in ordinary language.

It is certainly true that the professional physicist needs a vast mathematical apparatus in order to solve efficiently the problems of quantum mechanics. (For example, the calculation of the helium stripping threshold wavelength described above was a mathematical *tour de force*.) But this is not, I believe,* because quantum mechanics itself is fundamentally difficult or mathematical. I believe instead that the root rules of quantum mechanics are in fact quite simple. (They are unfamiliar and unexpected, but nevertheless simple.) When these rules are applied to particular situations, they are used over and over again and therefore the *applications* are complicated. An analogy helps explain this distinction. The rules of chess are very simple: they can be written on a single page of paper. But when these rules are applied to particular situations they are used over and over again and result in a complicated game: the applications of the chess rules fill a library.

Indeed, can any fundamental theory be highly mathematical? Electrons know how to obey quantum mechanics, and electrons can neither add nor subtract, much less solve partial differential equations! If something as simple-minded as an electron can understand quantum mechanics, then certainly something as wonderfully complex as the reader of this book can understand it too.

* Not everyone agrees with me.