

The Newtonian revolution

With illustrations of the transformation
of scientific ideas

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CONTENTS

<i>Preface</i>	<i>page ix</i>
PART ONE: THE NEWTONIAN REVOLUTION AND THE NEWTONIAN STYLE	
1 The Newtonian revolution in science	3
1.1. <i>Some basic features of the Scientific Revolution</i>	3
1.2. <i>A Newtonian revolution in science: the varieties of Newtonian science</i>	9
1.3. <i>Mathematics in the new science (1): a world of numbers</i>	17
1.4. <i>Mathematics in the new science (2): exact laws of nature and the hierarchy of causes</i>	21
1.5. <i>Causal mathematical science in the Scientific Revolution</i>	33
2 Revolution in science and the Newtonian revolution as historical concepts	39
2.1. <i>The concept of revolution</i>	39
2.2. <i>The introduction of the concept of revolution to describe scientific progress</i>	42
2.3. <i>The Newtonian revolution in the sciences</i>	49
3 The Newtonian revolution and the Newtonian style	52
3.1. <i>Some basic features of Newtonian exact science: mathematics and the disciplined creative imagination</i>	52
3.2. <i>Mathematics and physical reality in Newton's exact science</i>	61
3.3. <i>Newton's use of imagined systems and mathematical constructs in the Principia</i>	68
3.4. <i>Gravitation and attraction: Huygens's reaction to the Principia</i>	79
3.5. <i>Newton's path from imagined systems or constructs and mathematical principles to natural philosophy: the System of the World</i>	83

Supplement to 3.5: <i>Newton's first version of his System of the World and his "mathematical way" in fact and fiction</i>	93
3.6. <i>Mathematical systems or constructs and the review of the Principia in the Journal des Sçavans</i>	96
3.7. <i>Newton's three-phase procedure in action: Newton's constructs compared to Descartes's models and to those in use today</i>	99
3.8. <i>Newton's third phase and its sequel: the cause of gravitation</i>	109
3.9. <i>The Newtonian revolution as seen by some of Newton's successors: Bailly, Maupertuis, Clairaut</i>	120
3.10. <i>The Newtonian revolution in the perspective of history</i>	127
Supplement to 3.10: <i>Newtonian style or Galilean style</i>	132
3.11. <i>Optics and the Newtonian style</i>	133
3.12. <i>The ongoing Newtonian revolution and the Newtonian style: mathematics and experience</i>	141
PART TWO: TRANSFORMATIONS OF SCIENTIFIC IDEAS	
4 The transformation of scientific ideas	157
4.1. <i>A Newtonian synthesis?</i>	157
4.2. <i>Transformations of scientific ideas</i>	162
4.3. <i>Some examples of the transformation of scientific ideas: Darwin and intraspecific competition, Franklin and the electrical fluid</i>	166
4.4. <i>Some transformations of ideas by Newton, primarily the transformation of impulsive forces into continually acting forces and the formulation of Newton's third law</i>	171
4.5. <i>Newtonian inertia as an example of successive transformations</i>	182
4.6. <i>Some general aspects of transformations</i>	194
4.7. <i>The transformation of experience</i>	203
4.8. <i>The uniqueness of scientific innovation: Freud on originality</i>	216
4.9. <i>Transformations and scientific revolutions</i>	218
5 Newton and Kepler's laws: stages of transformation leading toward universal gravitation	222
5.1. <i>Kepler's laws and Newtonian principles</i>	222
5.2. <i>The status of Kepler's laws in Newton's day</i>	224
5.3. <i>Newton's early thoughts on orbital motion and Kepler's third law</i>	229
Supplement to 5.3. <i>An early computation of the moon's 'endeavour of receding' and a planetary inverse-square law</i>	238

5.4. <i>Newton and dynamical astronomy in the years before 1684: correspondence with Hooke in 1679–1680</i>	241
5.5. <i>The Newtonian discovery of the dynamical significance of Kepler's area law: the concept of force</i>	248
5.6. <i>From Kepler's laws to universal gravitation</i>	258
5.7. <i>The role of mass in Newtonian celestial mechanics</i>	271
5.8. <i>Kepler's laws, the motion of the moon, the Principia, and the Newtonian Scientific Revolution</i>	273
<i>Supplement: History of the concept of transformation: a personal account</i>	280
<i>Notes</i>	290
<i>Bibliography</i>	361
<i>Index</i>	397

The Newtonian revolution in science

1.1 *Some basic features of the Scientific Revolution*

A study of the Newtonian revolution in science rests on the fundamental assumption that revolutions actually occur in science. A correlative assumption must be that the achievements of Isaac Newton were of such a kind or magnitude as to constitute a revolution that may be set apart from other scientific revolutions of the sixteenth and seventeenth centuries. At once we are apt to be plunged deep into controversy. Although few expressions are more commonly used in writing about science than “scientific revolution”, there is a continuing debate as to the propriety of applying the concept and term “revolution” to scientific change.¹ There is, furthermore, a wide difference of opinion as to what may constitute a revolution. And although almost all historians would agree that a genuine alteration of an exceptionally radical nature (*the Scientific Revolution*²) occurred in the sciences at some time between the late fifteenth (or early sixteenth) century and the end of the seventeenth century, the question of exactly when this revolution occurred arouses as much scholarly disagreement as the cognate question of precisely what it was. Some scholars would place its origins in 1543, the year of publication of both Vesalius’s great work on the fabric of the human body and Copernicus’s treatise on the revolutions of the celestial spheres (Copernicus, 1543; Vesalius, 1543). Others would have the revolution be inaugurated by Galileo, possibly in concert with Kepler, while yet others would see Descartes as the true prime revolutionary. Contrariwise, a whole school of historians declare that many of the most significant features of the so-called Galilean revolution had emerged during the late Middle Ages.³

A historical analysis of the Newtonian revolution in science does

not, however, require participation in the current philosophical and sociological debates on these issues. For the fact of the matter is that the concept of revolution in science—in the sense in which we would understand this term nowadays—arose during Newton's day and was applied (see §2.2) first to a part of mathematics in which he made his greatest contribution, the calculus, and then to his work in celestial mechanics. Accordingly, the historian's task may legitimately be restricted to determining what features of Newton's science seemed so extraordinary in the age of Newton as to earn the designation of revolution. There is no necessity to inquire here into the various meanings of the term "revolution" and to adjudge on the basis of each such meaning the correctness of referring to a Newtonian revolution in the sciences.

The new science that took form during the seventeenth century may be distinguished by both external and internal criteria from the science and the philosophical study or contemplation of nature of the antecedent periods. Such an external criterion is the emergence in the seventeenth century of a scientific "community": individuals linked together by more or less common aims and methods, and dedicated to the finding of new knowledge about the external world of nature and of man that would be consonant with—and, accordingly, testable by—experience in the form of direct experiment and controlled observation. The existence of such a scientific community was characterized by the organization of scientific men into permanent formal societies, chiefly along national lines, with some degree of patronage or support by the state.⁴ The primary goal of such societies was the improvement of "natural knowledge".⁵ One way by which they sought to gain that end was through communication; thus the seventeenth century witnessed the establishment of scientific and learned journals, often the organs of scientific societies, including the *Philosophical Transactions* of the Royal Society of London, the *Journal des Sçavans*, and the *Acta eruditorum* of Leipzig.⁶ Another visible sign of the existence of a "new science" was the founding of research institutions, such as the Royal Greenwich Observatory, which celebrated its three-hundredth birthday in 1975. Newton's scientific career exhibits aspects of these several manifestations of the new science and the scientific community. He depended on the Astronomer Royal, John Flamsteed, for observational evidence that Jupiter might perturb the orbital motion of Saturn near conjunction and later

needed lunar positions from Flamsteed at the Greenwich Observatory in order to test and to advance his lunar theory, especially in the 1690s. His first scientific publication was his famous article on light and colors, which appeared in the pages of the *Philosophical Transactions*; his *Principia* was officially published by the Royal Society, of which he became president in 1703 (an office he kept until his death in 1727). While the Royal Society was thus of great importance in Newton's scientific life, it cannot be said that his activities in relation to that organization or its journal were in any way revolutionary.

The signs of the revolution can also be seen in internal aspects of science: aims, methods, results. Bacon and Descartes agreed on one aim of the new science, that the fruits of scientific investigation would be the improvement of man's condition here on earth:⁷ agriculture, medicine, navigation and transportation, communication, warfare, manufacturing, mining.⁸ Many scientists of the seventeenth century held to an older point of view, that the pursuit of scientific understanding of nature was practical insofar as it might advance man's comprehension of the divine wisdom and power. Science was traditionally practical in serving the cause of religion; but a revolutionary feature of the new science was the additional pragmatic goal of bettering everyday life here and now through applied science. The conviction that had been developing in the sixteenth and seventeenth centuries, that a true goal of the search for scientific truth must be to affect the material conditions of life, was then strong and widely shared, and constituted a novel and even a characteristic feature of the new science.

Newton often declared his conviction as to the older of these practicalities, as when he wrote to Bentley about his satisfaction in having advanced the cause of true religion by his scientific discoveries. Five years after the publication of his *Principia*, he wrote to Bentley that while composing the *Principia* ('my Treatise about our system'), 'I had an eye upon such Principles as might work with considering Men, for the Belief of a Deity' (Newton, 1958, p. 280; 1959-1977, vol. 3, p. 233). About two decades later, in 1713, he declared in the concluding general scholium to the *Principia* that the system of the world 'could not have arisen without the design and dominion of an intelligent and powerful being'. Newton was probably committed to some degree to the new practicality; at least he served as advisor to the official group concerned with the problem

of finding methods of determining the longitude at sea. Yet it was not Newton himself, but other scientists such as Halley, who attempted to link the Newtonian lunar theory with the needs of navigators, and the only major practical innovation that he produced was an instrument for science (the reflecting telescope) rather than inventions for man's more mundane needs.⁹

Another feature of the revolution was the attention to method. The attempts to codify method—by such diverse figures as Descartes, Bacon, Huygens, Hooke, Boyle, and Newton—signify that discoveries were to be made by applying a new tool of inquiry (a *novum organum*, as Bacon put it) that would direct the mind unerringly to the uncovering of nature's secrets. The new method was largely experimental, and has been said to have been based on induction; it also was quantitative and not merely observational and so could lead to mathematical laws and principles. I believe that the seventeenth-century evaluation of the importance of method was directly related to the role of experience (experiment and observation) in the new science. For it seems to have been a tacit postulate that any reasonably skilled man or woman should be able to reproduce an experiment or observation, provided that the report of that experiment or observation was given honestly and in sufficient detail. A consequence of this postulate was that anyone who understood the true methods of scientific enquiry and had acquired the necessary skill to make experiments and observations could have made the discovery in the first instance—provided, of course, that he had been gifted with the wit and insight to ask the right questions.¹⁰

This experimental or experiential feature of the new science shows itself also in the habit that arose of beginning an enquiry by repeating or reproducing an experiment or observation that had come to one's attention through a rumor or an oral or written report. When Galileo heard of a Dutch optical invention that enabled an observer to see distant objects as clearly as if they were close at hand, he at once set himself to reconstructing such an instrument.¹¹ Newton relates how he had bought a prism 'to try therewith the celebrated *Phaenomena* of *Colours*'.¹² From that day to this, woe betide any investigator whose experiments and observations could not be reproduced, or which were reported falsely; this attitude was based upon a fundamental conviction that nature's occurrences are constant and reproducible, thus subject

to universal laws. This twin requirement of performability and reproducibility imposed a code of honesty and integrity upon the scientific community that is itself yet another distinguishing feature of the new science.

The empirical aspect of the new science was just as significant with respect to the results achieved as with respect to the aims and methods. The law of falling bodies, put forth by Galileo, describes how real bodies actually fall on this earth—due consideration being given to the difference between the ideal case of a vacuum and the realities of an air-filled world, with winds, air resistance, and the effects of spin. Some of the laws of uniform and accelerated motion announced by Galileo can be found in the writings of certain late medieval philosopher-scientists, but the latter (with a single known exception of no real importance¹³) never even asked whether these laws might possibly correspond to any real or observable motions in the external world. In the new science, laws which do not apply to the world of observation and experiment could have no real significance, save as mathematical exercises. This point of view is clearly enunciated by Galileo in the introduction of the subject of 'naturally accelerated motion', in his *Two New Sciences* (1638). Galileo states the aim of his research to have been 'to seek out and clarify the definition that best agrees with that [accelerated motion] which nature employs' (Galileo, 1974, p. 153; 1890–1909, vol. 8, p. 197). From his point of view, there is nothing 'wrong with inventing at pleasure some kind of motion and theorizing about its consequent properties, in the way that some men have derived spiral and conchoidal lines from certain motions, though nature makes no use of these [paths]'. But this is different from studying motion in nature, for in exploring phenomena of the real external world, a definition is to be sought that accords with nature as revealed by experience:

But since nature does employ a certain kind of acceleration for descending heavy things, we decided to look into their properties so that we might be sure that the definition of accelerated motion which we are about to adduce agrees with the essence of naturally accelerated motion. And at length, after continual agitation of mind, we are confident that this has been found, chiefly for the very powerful reason that the essentials successively demonstrated by us correspond to, and are seen to be in

agreement with, that which physical experiments
[*naturalia experimenta*] show forth to the senses [ibid.].

Galileo's procedure is likened by him to having 'been led by the hand to the investigation of naturally accelerated motion by consideration of the custom and procedure of nature herself'.

Like Galileo, Newton the physicist saw the primary importance of concepts and rules or laws that relate to (or arise directly from) experience. But Newton the mathematician could not help but be interested in other possibilities. Recognizing that certain relations are of physical significance (as that 'the periodic times are as the $3/2$ power of the radii', or Kepler's third law), his mind leaped at once to the more universal condition (as that 'the periodic time is as any power R^n of the radius R ').¹⁴ Though Newton was willing to explore the mathematical consequences of attractions of spheres according to any rational function of the distance, he concentrated on the powers of index 1 and -2 since they are the ones that occur in nature: the power of index 1 of the distance from the center applies to a particle within a solid sphere and the power of index -2 to a particle outside either a hollow or solid sphere.¹⁵ It was his aim, in the *Principia*, to show that the abstract or 'mathematical principles' of the first two books could be applied to the phenomenologically revealed world, an assignment which he undertook in the third book. To do so, after Galileo, Kepler, Descartes, and Huygens, was not in itself revolutionary, although the scope of the *Principia* and the degree of confirmed application could well be so designated and thus be integral to the Newtonian revolution in science.

An excessive insistence on an out-and-out empirical foundation of seventeenth-century science has often led scholars to exaggerations.¹⁶ The scientists of that age did not demand that each and every statement be put to the test of experiment or observation, or even have such a capability, a condition that would effectively have blocked the production of scientific knowledge as we know it. But there was an insistence that the goal of science was to understand the real external world, and that this required the possibility of predicting testable results and retrodicting the data of actual experience: the accumulated results of experiment and controlled observation. This continual growth of factual knowledge garnered from the researches and observations made all over the world, paralleled by an equal and continual advance of understanding,

was another major aspect of the new science, and has been a distinguishing characteristic of the whole scientific enterprise ever since. Newton certainly made great additions to the stock of knowledge. In the variety and fundamental quality of these contributions we may see the distinguishing mark of his great creative genius, but this is something distinct from having created a revolution.

1.2 *A Newtonian revolution in science: the varieties of Newtonian science*

In the sciences, Newton is known for his contributions to pure and applied mathematics, his work in the general area of optics, his experiments and speculations relating to theory of matter and chemistry (including alchemy), and his systematization of rational mechanics (dynamics) and his celestial dynamics (including the Newtonian “system of the world”). Even a modest portion of these achievements would have sufficed to earn him an unquestioned place among the scientific immortals. In his own day (as we shall see below in Ch. 2), the word “revolution” began to be applied to the sciences in the sense of a radical change; one of the first areas in which such a revolution was seen to have occurred was in the discovery or invention of the calculus: a revolution in mathematics.¹ There is also evidence aplenty that in the age of Newton and afterwards, his *Principia* was conceived to have ushered in a revolution in the physical sciences. And it is precisely this revolution whose characteristic features I aim to elucidate.

Newton’s studies of chemistry and theory of matter yielded certain useful results² and numerous speculations. The latter were chiefly revealed in the queries at the end of the *Opticks*, especially the later ones,³ and in such a tract as the *De natura acidorum*.⁴ The significance of these writings and their influence have been aggrandized (from Newton’s day to ours) by the extraordinary place in science held by their author. At best, they are incomplete and programmatic and—in a sense—they chart out a possible revolution, but a revolution never achieved by Newton nor ever realized along the lines that he set down. Newton’s program and suggestions had a notable influence on the science of the eighteenth century, particularly the development of theories of heat and electricity (with their subtle elastic fluids) (cf. Cohen, 1956, Ch. 7, 8). Newton had a number of brilliant insights into the structure of matter and the process of chemical reaction, but the true revolution in chem-

istry did not come into being until the work of Lavoisier, which was not directly Newtonian (see Guerlac, 1975).

The main thrust of Newton's views on matter was the hope of deriving 'the rest of the phenomena of nature by the same kind of reasoning from mechanical principles' that had served in deducing 'the motions of the planets, the comets, the moon, and the sea'. He was convinced that all such phenomena, as he said in the preface (1686) to the first edition of the *Principia*, 'may depend upon certain forces by which the particles of bodies . . . are either mutually impelled [attracted] toward one another so as to cohere in regular figures' or 'are repelled and recede from one another'.⁵ In this way, as he put it on another occasion, the analogy of nature would be complete: 'Whatever reasoning holds for greater motions, should hold for lesser ones as well. The former depend upon the greater attractive forces of larger bodies, and I suspect that the latter depend upon the lesser forces, as yet unobserved, of insensible particles'. In short, Newton would have nature be thus 'exceedingly simple and conformable to herself'.⁶ This particular program was a conspicuous failure. Yet it was novel and can even be said to have had revolutionary features, so that it may at best represent a failed (or at least a never-achieved) revolution. But since we are concerned here with a positive Newtonian revolution, Newton's hope to develop a micro-mechanics analogous to his successful macro-mechanics is not our main concern. We cannot wholly neglect this topic, however, since it has been alleged that Newton's mode of attack on the physics of gross bodies and his supreme success in celestial mechanics was the product of his investigations of short-range forces, despite the fact that Newton himself said (and said repeatedly) that it was his success in the area of gravitation that led him to believe that the forces of particles could be developed in the same style. R. S. Westfall (1972, 1975) would not even stop there, but would have the 'forces of attraction between particles of matter', and also 'gravitational attraction which was probably the last one [of such forces] to appear', be 'primarily the offspring of alchemical active principles'. This particular thesis is intriguing in that it would give a unity to Newton's intellectual endeavor; but I do not believe it can be established by direct evidence (see Whiteside, 1977). In any event, Newton's unpublished papers on alchemy and his published (and unpublished) papers on chemistry and theory of matter hardly merit the appellation of "revolution",

in the sense of the radical influence on the advance of science that was exerted by the *Principia*.

In optics, the science of light and colors, Newton's contributions were outstanding. But his published work on 'The Reflections, Refractions, Inflexions [i.e., diffraction] & Colours of Light', as the *Opticks* was subtitled, was not revolutionary in the sense that the *Principia* was. Perhaps this was a result of the fact that the papers and book on optics published by Newton in his lifetime do not boldly display the mathematical properties of forces acting (as he thought) in the production of dispersion and other optical phenomena, although a hint of a mathematical model in the Newtonian style is given in passing in the *Opticks* (see §3.11) and a model is developed more fully in sect. 14 of bk. one of the *Principia*. Newton's first published paper was on optics, specifically on his prismatic experiments relating to dispersion and the composition of sunlight and the nature of color. These results were expanded in his *Opticks* (1704; Latin ed. 1706; second English ed. 1717/1718), which also contains his experiments and conclusions on other aspects of optics, including a large variety of what are known today as diffraction and interference phenomena (some of which Newton called the "inflexion" of light). By quantitative experiment and measurement he explored the cause of the rainbow, the formation of "Newton's rings" in sunlight and in monochromatic light, the colors and other phenomena produced by thin and thick "plates", and a host of other optical effects.⁷ He explained how bodies exhibit colors in relation to the type of illumination and their selective powers of absorption and transmission or reflection of different colors. The *Opticks*, even apart from the queries, is a brilliant display of the experimenter's art, where (as Andrade, 1947, p. 12, put it so well) we may see Newton's 'pleasure in shaping'. Some of his measurements were so precise that a century later they yielded to Thomas Young the correct values, to within less than 1 percent, of the wavelengths of light of different colors.⁸ Often cited as a model of how to perform quantitative experiments and how to analyze a difficult problem by experiment,⁹ Newton's studies of light and color and his *Opticks* nevertheless did not create a revolution and were not ever considered as revolutionary in the age of Newton or afterwards. In this sense, the *Opticks* was not epochal.

From the point of view of the Newtonian revolution in science,

however, there is one very significant aspect of the *Opticks*: the fact that in it Newton developed the most complete public statement he ever made of his philosophy of science or of his conception of the experimental scientific method. This methodological declaration has, in fact, been a source of some confusion ever since, because it has been read as if it applies to all of Newton's work, including the *Principia*.¹⁰ The final paragraph of qu. 28 of the *Opticks* begins by discussing the rejection of any 'dense Fluid' supposed to fill all space, and then castigates 'Later Philosophers' (i.e., Cartesians and Leibnizians) for 'feigning Hypotheses for explaining all things mechanically, and referring other Causes to Metaphysicks'. Newton asserts, however, that 'the main Business of natural Philosophy is to argue from Phaenomena without feigning Hypotheses, and to deduce Causes from Effects, till we come to the very first Cause, which certainly is not mechanical'.¹¹ Not only is the main assignment 'to unfold the Mechanism of the World', but it is to 'resolve' such questions as: 'What is there in places almost empty of Matter . . . ?' 'Whence is it that Nature doth nothing in vain; and whence arises all that Order and Beauty which we see in the World?' 'What 'hinders the fix'd Stars from falling upon one another?' 'Was the Eye contrived without Skill in Opticks, and the Ear without Knowledge of Sounds?' or 'How do the Motions of the Body follow from the Will, and whence is the Instinct in Animals?'

In qu. 31, Newton states his general principles of analysis and synthesis, or resolution and composition, and the method of induction:

As in Mathematicks, so in Natural Philosophy, the Investigation of difficult Things by the Method of Analysis, ought ever to precede the Method of Composition. This Analysis consists in making Experiments and Observations, and in drawing general Conclusions from them by Induction, and admitting of no Objections against the Conclusions, but such as are taken from Experiments, or other certain Truths. For Hypotheses are not to be regarded in experimental Philosophy. And although the arguing from Experiments and Observations by Induction be no Demonstration of general Conclusions; yet it is the best way of arguing which the Nature of Things admits of, and may be looked upon as so much the stronger, by how much the Induction is more general.

Analysis thus enables us to

proceed from Compounds to Ingredients, and from Motions to the Forces producing them; and in general, from Effects to their Causes, and from particular Causes to more general ones, till the Argument end in the most general.

This method of analysis is then compared to synthesis or composition:

And the Synthesis consists in assuming the Causes discover'd, and establish'd as Principles, and by them explaining the Phaenomena proceeding from them, and proving the Explanations.¹²

The lengthy paragraph embodying the foregoing three extracts is one of the most often quoted statements made by Newton, rivaled only by the concluding General Scholium of the *Principia*, with its noted expression: *Hypotheses non fingo*.

Newton would have us believe that he had himself followed this "scenario":¹³ first, to reveal by "analysis" some simple results that were generalized by induction, thus proceeding from effects to causes and from particular causes to general causes; then, on the basis of these causes considered as principles, to explain by "synthesis" the phenomena of observation and experiment that may be derived or deduced from them, 'proving the Explanations'. Of the latter, Newton says that he has given an 'Instance . . . in the End of the first Book' where the 'Discoveries being proved [by experiment] may be assumed in the Method of Composition for explaining the Phaenomena arising from them'. An example, occurring at the end of bk. one, pt. 2, is props. 8–11, with which pt. 2 concludes. Prop. 8 reads: 'By the discovered Properties of Light to explain the Colours made by Prisms'. Props. 9–10 also begin: 'By the discovered Properties of Light to explain . . .', followed (prop. 9) by 'the Rain-bow' and (prop. 10) by 'the permanent Colours of Natural Bodies'. Then, the concluding prop. 11 reads: 'By mixing coloured Lights to compound a beam of Light of the same Colour and Nature with a beam of the Sun's direct Light'.

The formal appearance of the *Opticks* might have suggested that it was a book of synthesis, rather than analysis, since it begins (bk. one, pt. 1) with a set of eight 'definitions' followed by eight 'axioms'. But the elucidation of the propositions that follow does

not make explicit reference to these axioms, and many of the individual propositions are established by a method plainly labeled 'The PROOF by Experiments'. Newton himself states clearly at the end of the final qu. 31 that in bks. one and two he has 'proceeded by . . . Analysis' and that in bk. three (apart from the queries) he has 'only begun the Analysis'. The structure of the *Opticks* is superficially similar to that of the *Principia*, for the *Principia* also starts out with a set of 'definitions' (again eight in number), followed by three 'axioms' (three 'axiomata sive leges motus'), upon which the propositions of the first two books are to be constructed (as in the model of Euclid's geometry). But then, in bk. three of the *Principia*, on the system of the world, an ancillary set of so-called 'phenomena' mediate the application of the mathematical results of bks. one and two to the motions and properties of the physical universe.¹⁴ Unlike the *Opticks*, the *Principia* does make use of the axioms and definitions.¹⁵ The confusing aspect of Newton's stated method of analysis and synthesis (or composition) in qu. 31 of the *Opticks* is that it is introduced by the sentence 'As in Mathematicks, so in Natural Philosophy . . .', which was present when this query first appeared (as qu. 23) in the Latin *Optice* in 1706, 'Quemadmodum in Mathematica, ita etiam in Physica . . .'. A careful study, however, shows that Newton's usage in experimental natural philosophy is just the reverse of the way "analysis" and "synthesis" (or "resolution" and "composition") have been traditionally used in relation to mathematics, and hence in the *Principia*—an aspect of Newton's philosophy of science that was fully understood by Dugald Stewart a century and a half ago but that has not been grasped by present-day commentators on Newton's scientific method, who would even see in the *Opticks* the same style that is to be found in the *Principia*¹⁶ (this point is discussed further in §3.11).

Newton's "method", as extracted from his *dicta* rather than his *opera*, has been summarized as follows: 'The main features of Newton's method, it seems, are: The rejection of hypotheses, the stress upon induction, the working sequence (induction precedes deduction), and the inclusion of metaphysical arguments in physics' (Turbayne, 1962, p. 45). Thus Colin Turbayne would have 'the deductive procedure' be a defining feature of Newton's 'mathematical way' and Descartes's '*more geometrico*' respectively: 'Descartes's "long chains of reasoning" were deductively linked. New-