200 Puzzling Physics Problems

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Contents

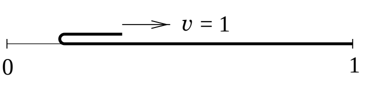
Problems

P1 Three small snails are each at a vertex of an equilateral triangle of side 60 cm. The first sets out towards the second, the second towards the third and the third towards the first, with a uniform speed of 5 cm min^{-1}. During their motion each of them always heads towards its respective target snail. How much time has elapsed, and what distance do the snails cover, before they meet? What is the equation of their paths? If the snails are considered as point-masses, how many times does each circle their ultimate meeting point?

P2 A small object is at rest on the edge of a horizontal table. It is pushed in such a way that it falls off the other side of the table, which is 1 m wide, after 2 s. Does the object have wheels?

P3 A boat can travel at a speed of 3 m s^{-1} on still water. A boatman wants to cross a river whilst covering the shortest possible distance. In what direction should he row with respect to the bank if the speed of the water is (i) 2 m s^{-1}, (ii) 4 m s^{-1}? Assume that the speed of the water is the same everywhere.

P4 A long, thin, pliable carpet is laid on the floor. One end of the carpet is bent back and then pulled backwards with constant unit velocity, just above the part of the carpet which is still at rest on the floor.

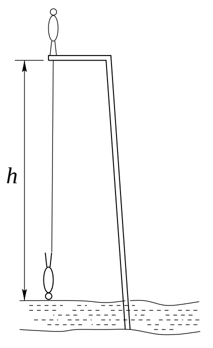


Find the speed of the centre of mass of the moving part. What is the minimum force needed to pull the moving part, if the carpet has unit length and unit mass?

P5 Four snails travel in uniform, rectilinear motion on a very large plane surface. The directions of their paths are random, (but not parallel, i.e. any two snails could meet), but no more than two snail paths can cross at any one point. Five of the $(4 \times 3)/2 = 6$ possible encounters have already occurred. Can we state with certainty that the sixth encounter will also occur?

P6 Two 20-g flatworms climb over a very thin wall, 10 cm high. One of the worms is 20 cm long, the other is wider and only 10 cm long. Which of them has done more work against gravity when half of it is over the top of the wall? What is the ratio of the amounts of work done by the two worms?

P7 A man of height $h_0 = 2$ m is bungee jumping from a platform situated a height $h = 25$ m above a lake. One end of an elastic rope is attached to his foot and the other end is fixed to the platform. He starts falling from rest in a vertical position.



The length and elastic properties of the rope are chosen so that his speed will have been reduced to zero at the instant when his head reaches the surface of the water. Ultimately the jumper is hanging from the rope, with his head 8 m above the water.

(i) Find the unstretched length of the rope.
(ii) Find the maximum speed and acceleration achieved during the jump.

P8 An iceberg is in the form of an upright regular pyramid of which 10 m shows above the water surface. Ignoring any induced motion of the water, find the period of small vertical oscillations of the berg. The density of ice is 900 kg m^{-3}.

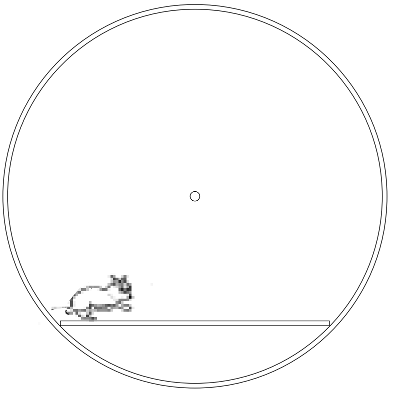
P9 The suspension springs of all four wheels of a car are identical. By how much does the body of the car (considered rigid) rise above each of the wheels when its right front wheel is parked on an 8-cm-high pavement? Does the result change when the car is parked with both right wheels on the pavement? Does the result depend on the number and positions of the people sitting in the car?

P10* In Victor Hugo's novel *les Misérables*, the main character Jean Valjean, an escaped prisoner, was noted for his ability to climb up the corner formed by the intersection of two vertical perpendicular walls. Find the minimum force with which he had to push on the walls whilst climbing. What is the minimum coefficient of static friction required for him to be able to perform such a feat?

P11 A sphere, made of two non-identical homogeneous hemispheres stuck together, is placed on a plane inclined at an angle of 30° to the horizontal. Can the sphere remain in equilibrium on the inclined plane?

P12 A small, elastic ball is dropped vertically onto a long plane inclined at an angle α to the horizontal. Is it true that the distances between consecutive bouncing points grow as in an arithmetic progression? Assume that collisions are perfectly elastic and that air resistance can be neglected.

P13 A small hamster is put into a circular wheel-cage, which has a frictionless central pivot. A horizontal platform is fixed to the wheel below the pivot. Initially, the hamster is at rest at one end of the platform.



When the platform is released the hamster starts running, but, because of the hamster's motion, the platform and wheel remain *stationary*. Determine how the hamster moves.

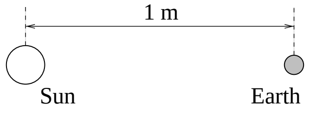
P14* A bicycle is supported so that it is prevented from falling sideways but can move forwards or backwards; its pedals are in their highest and lowest positions. A student crouches beside the bicycle and applies a horizontal force, directed towards the back wheel, to the lower pedal.

(i) Which way does the bicycle move?

(ii) Does the chain-wheel rotate in the same or opposite sense as the rear wheel?

(iii) Which way does the lower pedal move relative to the ground?

P15 If the solar system were proportionally reduced so that the average distance between the Sun and the Earth were 1 m, how long would a year be? Take the density of matter to be unchanged.



P16 If the mass of each of the members of a binary star were the same as that of the Sun, and their distance apart were equal to the Sun–Earth distance, what would be their period of revolution?

P17 (i) What is the minimum launch speed required to put a satellite into a circular orbit?

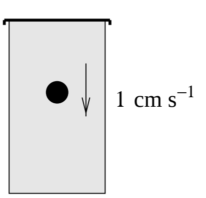
(ii) How many times higher is the energy required to launch a satellite into a polar orbit than that necessary to put it into an Equatorial one?

(iii) What initial speed must a space probe have if it is to leave the gravitational field of the Earth?

(iv) Which requires a higher initial energy for the space probe – leaving the solar system or hitting the Sun?

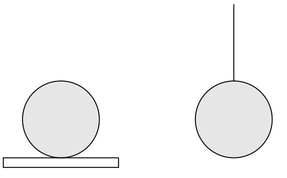
P18 A rocket is intended to leave the Earth's gravitational field. The fuel in its main engine is a little less than the amount that is necessary, and an auxiliary engine, only capable of operating for a short time, has to be used as well. When is it best to switch on the auxiliary engine: at take-off, or when the rocket has nearly stopped with respect to the Earth, or does it not matter?

P19 A steel ball with a volume of 1 cm^3 is sinking at a speed of 1 cm s^{-1} in a closed jar filled with honey. What is the momentum of the honey if its density is 2 g cm^{-3}?



P20 A gas of temperature T is enclosed in a container whose walls are (initially) at temperature T_1. Does the gas exert a higher pressure on the walls of the container when $T_1 < T$ or when $T_1 > T$?

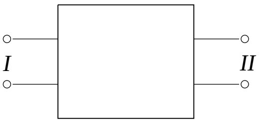
P21* Consider two identical iron spheres, one of which lies on a thermally insulating plate, whilst the other hangs from an insulating thread.



Equal amounts of heat are given to the two spheres. Which will have the higher temperature?

P22 Two (non-physics) students, A and B, living in neighbouring college rooms, decided to economise by connecting their ceiling lights in series. They agreed that each would install a 100-W bulb in their own rooms and that they would pay equal shares of the electricity bill. However, both decided to try to get better lighting at the other's expense; A installed a 200-W bulb and B installed a 50-W bulb. Which student subsequently failed the end-of-term examinations?

P23 If a battery of voltage V is connected across terminals *I* of the black box shown in the figure, a voltmeter connected to terminals *II* gives a reading of $V/2$; while if the battery is connected to terminals *II*, a voltmeter across terminals *I* reads V.

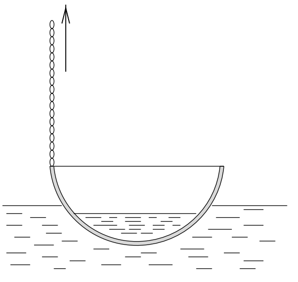


The black box contains only passive circuit elements. What are they?

P24 A bucket of water is suspended from a fixed point by a rope. The bucket is set in motion and the system swings as a pendulum. However, the bucket leaks and the water slowly flows out of the bottom of it. How does the period of the swinging motion change as the water is lost?

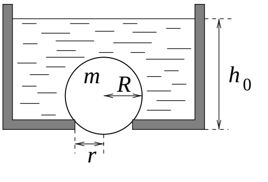
P25 An empty cylindrical beaker of mass 100 g, radius 30 mm and negligible wall thickness, has its centre of gravity 100 mm above its base. To what depth should it be filled with water so as to make it as stable as possible?

P26 Fish soup is prepared in a hemispherical copper bowl of diameter 40 cm. The bowl is placed into the water of a lake to cool down and floats with 10 cm of its depth immersed.



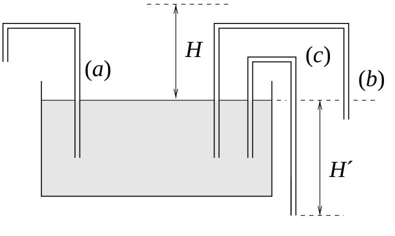
A point on the rim of the bowl is pulled upwards through 10 cm, by a chain fastened to it. Does water flow into the bowl?

P27 A circular hole of radius r at the bottom of an initially full water container is sealed by a ball of mass m and radius $R\,(> r)$. The depth of the water is now slowly reduced, and when it reaches a certain value, h_0, the ball rises out of the hole. Find h_0.



P28 Soap bubbles filled with helium float in air. Which has the greater mass – the wall of a bubble or the gas enclosed within it?

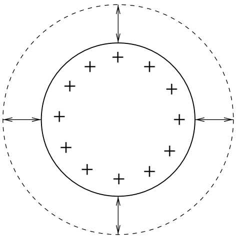
P29 Water which wets the walls of a vertical capillary tube rises to a height H within it. Three 'gallows', (a), (b) and (c), are made from the same tubing, and one end of each is placed into a large dish filled with water, as shown in the figure.



Does the water flow out at the other ends of the capillary tubes?

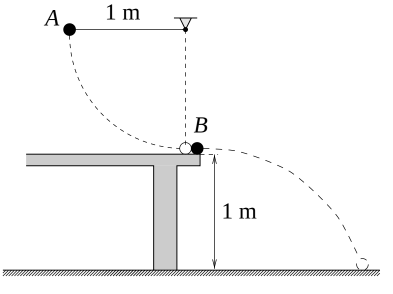
P30 A charged spherical capacitor slowly discharges as a result of the slight conductivity of the dielectric between its concentric plates. What are the magnitude and direction of the magnetic field caused by the resulting electric current?

P31 An electrically charged conducting sphere 'pulses' radially, i.e. its radius changes periodically with a fixed amplitude (*see figure*). The charges on its surface – acting as many dipole antennae – emit electromagnetic radiation. What is the net pattern of radiation from the sphere?



P32* How high would the male world-record holder jump (at an indoor competition!) on the Moon?

P33 A small steel ball B is at rest on the edge of a table of height 1 m. Another steel ball A, used as the bob of a metre-long simple pendulum, is released from rest with the pendulum suspension horizontal, and swings against B as shown in the figure. The masses of the balls are identical and the collision is elastic.



Considering the motion of B only up until the moment it first hits the ground:

(i) Which ball is in motion for the longer time?
(ii) Which ball covers the greater distance?

P34 A small bob is fixed to one end of a string of length 50 cm. As a consequence of the appropriate forced motion of the other end of the string, the bob moves in a vertical circle of radius 50 cm with a uniform speed of 3.0 m s^{-1}. Plot, at 15° intervals on the circular path, the trajectories of both ends of the string, indicating on each the points belonging together.

P35 A point P is located above an inclined plane. It is possible to reach the plane by sliding under gravity down a straight frictionless wire, joining P to some point P' on the plane. How should P' be chosen so as to minimise the time taken?

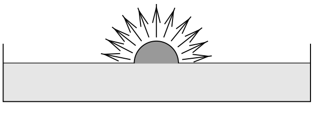
P36 The minute hand of a church clock is twice as long as the hour hand. At what time after midnight does the end of the minute hand move away from the end of the hour hand at the fastest rate?

P37 What is the maximum angle to the horizontal at which a stone can be thrown and always be moving away from the thrower?

P38* A tree-trunk of diameter 20 cm lies in a horizontal field. A lazy grasshopper wants to jump over the trunk. Find the minimum take-off speed of the grasshopper that will suffice. (Air resistance is negligible.)

P39* A straight uniform rigid hair lies on a smooth table; at each end of the hair sits a flea. Show that if the mass M of the hair is not too great relative to that m of each of the fleas, they can, by simultaneous jumps with the same speed and angle of take-off, exchange ends without colliding in mid-air.

P40 A fountain consists of a small hemispherical rose (sprayer) which lies on the surface of the water in a basin, as illustrated in the figure. The rose has many evenly distributed small holes in it, through which water spurts at the same speed in all directions.



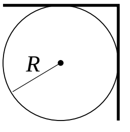
What is the shape of the water 'bell' formed by the jets?

P41 A particle of mass m carries an electric charge Q and is subject to the combined action of gravity and a uniform horizontal electric field of strength E. It is projected with speed v in the vertical plane parallel to the field and at an angle θ to the horizontal. What is the maximum distance the particle can travel horizontally before it is next level with its starting point?

P42** A uniform rod of mass m and length ℓ is supported horizontally at its ends by my two forefingers. Whilst I am slowly bringing my fingers together to meet under the centre of the rod, it slides on either one or other of them. How much work do I have to do during the process if the coefficient of static friction is μ_{stat}, and that of kinetic friction is μ_{kin} ($\mu_{\text{kin}} \leq \mu_{\text{stat}}$)?

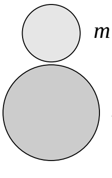
P43 Four identical bricks are placed on top of each other at the edge of a table. Is it possible to slide them horizontally across each other in such a way that the projection of the topmost one is completely outside the table? What is the theoretical limit to the displacement of the topmost brick if the number of bricks is arbitrarily increased?

P44 A plate, bent at right angles along its centre line, is placed onto a horizontal fixed cylinder of radius R as shown in the figure.



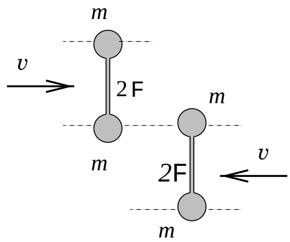
How large does the coefficient of static friction between the cylinder and the plate need to be if the plate is not to slip off the cylinder?

P45 Two elastic balls of masses m_1 and m_2 are placed on top of each other (with a small gap between them) and then dropped onto the ground. What is the ratio m_1/m_2, for which the upper ball ultimately receives the largest possible fraction of the total energy? What ratio of masses is necessary if the upper ball is to bounce as high as possible?



P46 An executive toy consists of three suspended steel balls of masses M, μ and m arranged in that order with their centres in a horizontal line. The ball of mass M is drawn aside in their common plane until its centre has been raised by h and is then released. If $M \neq m$ and all collisions are elastic, how must μ be chosen so that the ball of mass m rises to the greatest possible height? What is this height? (Neglect multiple collisions.)

P47 Two identical dumb-bells move towards each other on a horizontal air-cushioned table, as shown in the figure. Each can be considered as two point masses m joined by a weightless rod of length 2ℓ. Initially, they are not rotating. Describe the motion of the dumb-bells after their elastic collision. Plot the speeds of the centres of mass of the dumb-bells as a function of time.



P48 Two small identical smooth blocks A and B are free to slide on a frozen lake. They are joined together by a light elastic rope of length $\sqrt{2}L$ which has the property that it stretches very little when the rope becomes taut. At time $t = 0$, A is at rest at $x = y = 0$ and B is at $x = L$, $y = 0$ moving in the positive y-direction with speed V. Determine the positions and velocities of A and B at times (i) $t = 2L/V$ and (ii) $t = 100L/V$.

P49* After a tap above an empty rectangular basin has been opened, the basin fills with water in a time T_1. After the tap has been closed, opening a plug-hole at the bottom of the basin empties it in a time T_2. What happens if both the tap and the plug-hole are open? What ratio of T_1/T_2 can cause the basin to overflow? As a specific case, let $T_1 = 3$ minutes and $T_2 = 2$ minutes.

P50 A cylindrical vessel of height h and radius a is two-thirds filled with liquid. It is rotated with constant angular velocity ω about its axis, which is vertical. Neglecting any surface tension effects, find an expression for the greatest angular velocity of rotation Ω for which the liquid does not spill over the edge of the vessel.

P51 Peter, who was standing by a racetrack, calculated that as one of the cars, in accelerating from rest to a speed of 100 km h^{-1}, used up x litres of fuel, it could increase its speed to 200 km h^{-1}, by using a further $3x$ litres of fuel. Peter, who has learned in physics that kinetic energy is proportional to the square of the speed, assumed that the energy content of the fuel was mainly converted into kinetic energy, i.e. he neglected air resistance and other types of friction.

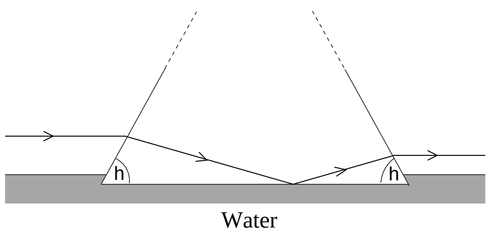
A railway runs by the racetrack. Paul, who also knows some physics, saw the start of the race from the window of a train travelling at a speed of 100 km h^{-1} in the opposite direction to that of the car. He reasoned as follows: since the car's speed increased from 100 to 200 km h^{-1} during the first stage, when the car accelerates from 200 to 300 km h^{-1} in the second stage, it will need $(300^2 - 200^2)/(200^2 - 100^2)\ x = (5/3)x$ litres of fuel.

Who is right, Peter or Paul?

P52 The distance between a screen and a light source lined up on an optical bench is 120 cm. When a lens is moved along the line joining them, sharp images of the source can be obtained at two lens positions; the (linear) size ratio of these two images is 1 : 9. What is the focal length of the lens? Which image is the brighter? Determine the ratio of the brightness values of the two images.

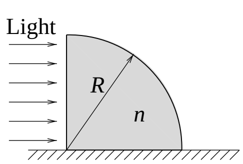
P53 A short-sighted person takes off his glasses and observes a fixed object through them, while moving the glasses away from his eyes. He is surprised to see that at first, the object looks smaller and smaller, but then becomes larger and larger. What is the reason for this?

P54 A glass prism whose cross-section is an isosceles triangle stands with its (horizontal) base in water; the angles that its two equal sides make with the base are each θ.



An incident ray of light, above and parallel to the water surface and perpendicular to the prism's axis, is internally reflected at the glass–water interface and subsequently re-emerges into the air. Taking the refractive indices of glass and water to be $\frac{3}{2}$ and $\frac{4}{3}$, respectively, show that θ must be at least 25.9°.

P55 A glass prism in the shape of a quarter-cylinder lies on a horizontal table. A uniform, horizontal light beam falls on its vertical plane surface, as shown in the figure.

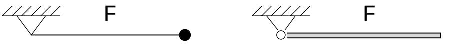


If the radius of the cylinder is $R = 5$ cm and the refractive index of the glass is $n = 1.5$, where, on the table beyond the cylinder, will a patch of light be found?

P56 How much brighter is sunlight than moonlight? The albedo (reflectivity) of the Moon is $\alpha = 0.07$.

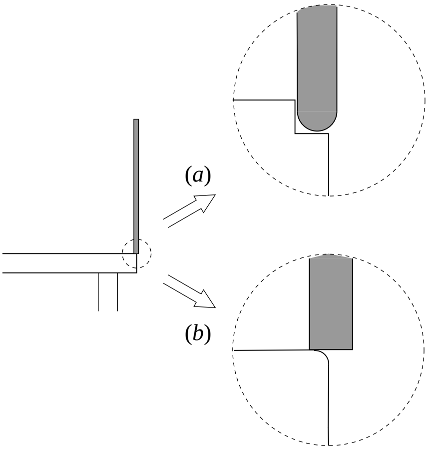
P57 Annie and her very tall boyfriend Andy like jogging together. They notice that when running they move at more or less the same speed, but that Andy is always faster when they are walking. How can this difference between running and walking be explained using physical arguments?

P58 A simple pendulum and a homogeneous rod pivoted at its end are released from horizontal positions. What is the ratio of their periods of swing if their lengths are identical?



P59* A helicopter can hover when the power output of its engine is P. A second helicopter is an exact copy of the first one, but its linear dimensions are half those of the original. What power output is needed to enable this second helicopter to hover?

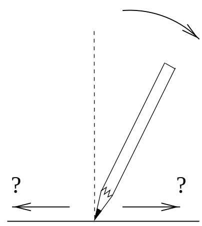
P60* A uniform rod is placed with one end on the edge of a table in a nearly vertical position and is then released from rest. Find the angle it makes with the vertical at the moment it loses contact with the table. Investigate the following two extreme cases:



(i) The edge of the table is smooth (friction is negligible) but has a small, single-step groove as shown in figure (*a*).

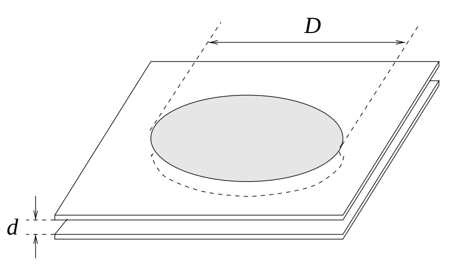
(ii) The edge of the table is rough (friction is large) and very sharp, which means that the radius of curvature of the edge is much smaller than the flat end-face of the rod. Half of the end-face protrudes beyond the table edge (*see figure* (*b*)), with the result that when it is released from rest the rod 'pivots' about the edge. The rod is much longer than its diameter.

P61** A pencil is placed vertically on a table with its point downwards. It is then released and tumbles over. How does the direction in which the point moves, relative to that in which the pencil falls, depend upon the coefficient of friction? Will the pencil point lose contact with the table (other than when the 'shoulder' of the pencil ultimately comes into contact with the table)?



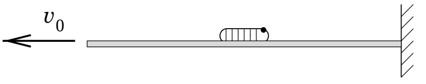
P62 Two soap bubbles of radii R_1 and R_2 are joined by a straw. Air goes from one bubble to the other (which one?) and a single bubble of radius R_3 is formed. What is the surface tension of the soap solution if the atmospheric pressure is p_0? Is measuring three such radii a suitable method for determining the surface tension of liquids?

P63 Water, which wets glass, is stuck between two parallel glass plates. The distance between the plates is d, and the diameter of the trapped water 'disc' is $D \gg d$.



What is the force acting between the plates?

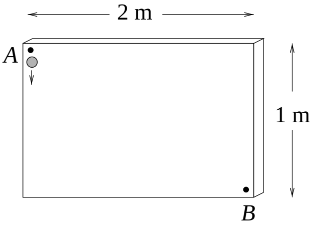
P64 A spider has fastened one end of a 'super-elastic' silk thread of length 1 m to a vertical wall. A small caterpillar is sitting somewhere on the thread.



The hungry spider, whilst not moving from its original position, starts pulling in the other end of the thread with uniform speed, $v_0 = 1$ cm s^{-1}. Meanwhile, the caterpillar starts fleeing towards the wall with a uniform speed of 1 mm s^{-1} with respect to the moving thread. Will the caterpillar reach the wall?

P65* How does the solution to the previous problem change if the spider does not sit in one place, but moves (away from the wall) taking with it the end of the thread?

P66 Nails are driven horizontally into a vertically placed drawing-board. As shown in the figure, a small steel ball is dropped from point A and reaches point B by bouncing elastically on the protruding nails (which are not shown in the figure).

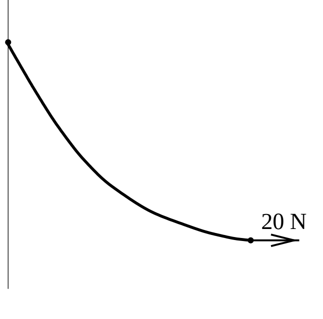


Is it possible to arrange the nails so that:

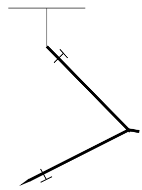
(i) The ball gets from point A to point B more quickly than if it had slid without friction down the shortest path, i.e. along the straight line AB?

(ii) The ball reaches point B in less than 0.4 s?

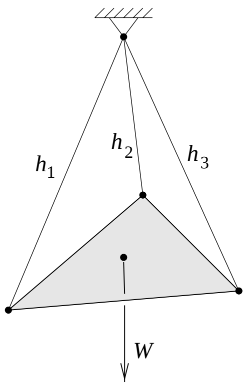
P67 One end of a rope is fixed to a vertical wall and the other end pulled by a horizontal force of 20 N. The shape of the flexible rope is shown in the figure. Find its mass.



P68 Find the angle to which a pair of compasses should be opened in order to have the pivot as elevated as possible when the compasses are suspended from a string attached to one of the points, as shown in the figure. Assume that the lengths of the compass arms are equal.

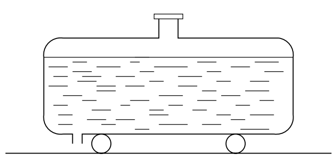


P69* Threads of lengths h_1, h_2 and h_3 are fastened to the vertices of a homogeneous triangular plate of weight W. The other ends of the threads are fastened to a common point, as shown in the figure.



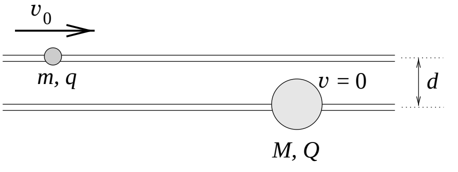
What is the tension in each thread, expressed in terms of the lengths of the threads and the weight of the plate?

P70* A tanker full of liquid is parked at rest on a horizontal road. The brake has not been applied, and it may be supposed that the tanker can move without friction.



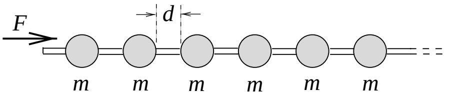
In which direction will the tanker move after the tap on the vertical outlet pipe, which is situated at the rear of the tanker, has been opened? Will the tanker continue to move in this direction?

P71 Two small beads slide without friction, one on each of two long, horizontal, parallel, fixed rods set a distance d apart. The masses of the beads are m and M, and they carry respective charges of q and Q. Initially, the larger mass M is at rest and the other one is far away approaching it at speed v_0.



Describe the subsequent motion of the beads.

P72* Beads of equal mass are strung at equal distances on a long, horizontal wire. The beads are initially at rest but can move without friction.

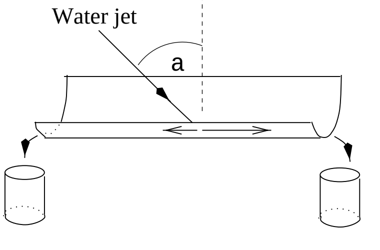


One of the beads is continuously accelerated (towards the right) by a constant force F. What are the speeds of the accelerated bead and the front of the 'shock wave', after a long time, if the collisions of the beads are:

(i) completely inelastic,
(ii) perfectly elastic?

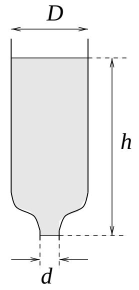
P73* A table and a large jug are placed on the platform of a weighing machine and a barrel of beer is placed on the table with its tap above the jug. Describe how the reading of the machine varies with time after the tap has been opened and the beer runs into the jug.

P74 A jet of water strikes a horizontal gutter of semicircular cross-section obliquely, as shown in the figure. The jet lies in the vertical plane that contains the centre-line of the gutter.



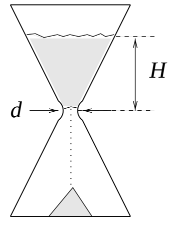
Calculate the ratio of the quantities of water flowing out at the two ends of the gutter as a function of the angle of incidence α of the jet.

P75* An open-topped vertical tube of diameter D is filled with water up to a height h. The narrow bottom-end of the tube, of diameter d, is closed by a stop as shown in the figure.

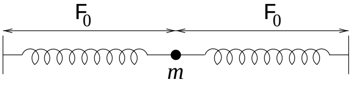


When the stop is removed, the water starts flowing out through the bottom orifice with approximate speed $v = \sqrt{2gh}$. However, this speed is reached by the liquid only after a certain time τ. Obtain an estimate of the order of magnitude of τ. What is the acceleration of the lowest layer of water at the moment when the stop is removed? Ignore viscous effects.

P76* Obtain a reasoned estimate of the time it takes for the sand to run down through an egg-timer. Use realistic data.

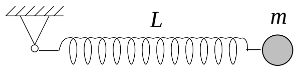


P77 A small bob joins two light unstretched, identical springs, anchored at their far ends and arranged along a straight line, as shown in the figure.



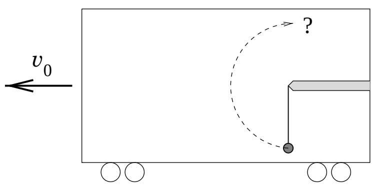
The bob is displaced in a direction perpendicular to the line of the springs by 1 cm and then released. The period of the ensuing vibration of the bob is 2 s. Find the period of the vibration if the bob were displaced by 2 cm before release. The unstretched length of the springs is $\ell_0 \gg 1$ cm, and gravity is to be ignored.

P78* One end of a light, weak spring, of unstretched length L and force constant k, is fixed to a pivot, and a body of mass m is attached to its other end. The spring is released from an unstretched, horizontal position, as in the figure.



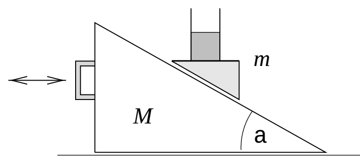
What is the length of the spring when it reaches a vertical position? (Describing a spring as weak implies that $mg \gg kL$, and that the tension in the spring is directly proportional to its extension at all times.)

P79* A heavy body of mass m hangs on a flexible thread in a railway carriage which moves at speed v_0 on a train-safety test track, as shown in the figure.



The carriage is brought to rest by a strong but uniform braking. Can the pendulum travel through 180°, so that the taut thread reaches the vertical?

P80** A glass partially filled with water is fastened to a wedge that slides, without friction, down a large plane inclined at an angle α as shown in the figure. The mass of the inclined plane is M, the combined mass of the wedge, the glass and the water is m.



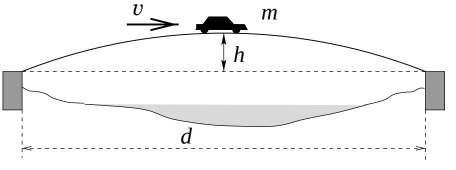
If there were no motion the water surface would be horizontal. What angle will it ultimately make with the inclined plane if

(i) the inclined plane is fixed,
(ii) the inclined plane can move freely in the horizontal direction?

Examine also the case in which $m \gg M$. What happens if the handle of the inclined plane is shaken in a periodic manner, but one that is such that it does not cause the wedge to rise off the plane?

P81** If someone found a motionless string reaching vertically up into the sky and hanging down nearly to the ground, should that person consider it as an evidence for UFOs, or could there be an 'Earthly' explanation in agreement with the well-known laws of physics? How long would the string need to be?

P82 There is a parabolic-shaped bridge across a river of width 100 m. The highest point of the bridge is 5 m above the level of the banks. A car of mass 1000 kg is crossing the bridge at a constant speed of 20 m s^{-1}.



Using the notation indicated in the figure, find the force exerted on the bridge by the car when it is:

(i) at the highest point of the bridge,

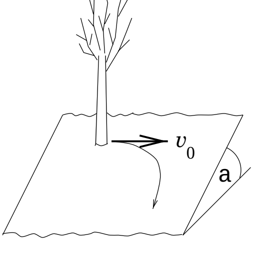
(ii) three-quarters of the way across.

(Ignore air resistance and take g as 10 m s^{-2}.)

P83 A point mass of 0.5 kg moving with a constant speed of 5 m s^{-1} on an elliptical track experiences an outward force of 10 N when at either endpoint of the major axis and a similar force of 1.25 N at each end of the minor axis. How long are the axes of the ellipse?

P84* A boatman sets off from one bank of a straight, uniform canal for a mark directly opposite the starting point. The speed of the water flowing in the canal is v everywhere. The boatman rows steadily at such a rate that, were there no current, the boat's speed would also be v. He always sets the boat's course in the direction of the mark, but the water carries him downstream. Fortunately he never tires! How far downstream does the water carry the boat? What trajectory does it follow with respect to the bank?

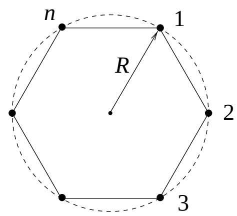
P85** Two children stand on a large, sloping hillside that can be considered as a plane. The ground is just sufficiently icy that a child would fall and slide downhill with a uniform speed as the result of receiving even the slightest impulse.



For fun, one of the children (leaning against a tree) pushes the other with a *horizontal* initial speed $v_0 = 1\ \mathrm{m\ s^{-1}}$. The latter slides down the slope with a velocity that changes in both magnitude and direction. What will be the child's final speed if air resistance is negligible and the frictional force is independent of the speed?

P86* Smugglers set off in a ship in a direction perpendicular to a straight shore and move at constant speed v. The coastguard's cutter is a distance a from the smugglers' ship and leaves the shore at the same time. The cutter always moves at a constant speed in the direction of the smugglers' ship and catches up with the criminals when at a distance a from the shore. How many times greater is the speed of the coastguard's cutter than that of the smugglers' ship?

P87 Point-masses of mass m are at rest at the corners of a regular n-gon, as illustrated in the figure for $n = 6$.



How does the system move if only gravitation acts between the bodies? How much time elapses before the bodies collide if $n = 2$, 3 and 10? Examine the limiting case when $n \gg 1$ and $m = M_0/n$, where M_0 is a given total mass.

P88 A rocket is launched from and returns to a spherical planet of radius R in such a way that its velocity vector on return is parallel to its launch vector. The angular separation at the centre of the planet between the launch and arrival points is θ. How long does the flight of the rocket take, if the period of a satellite flying around the planet just above its surface is T_0? What is the maximum distance of the rocket above the surface of the planet? Consider whether your analysis also applies to the limiting case of $\theta \to 0$.

P89** Two identical small magnets of moment μ are glued to opposite ends of a wooden rod of length L, one labelled C, parallel to the rod, and the other labelled D, perpendicular to it.