BOSE–EINSTEIN CONDENSATION

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Introduction: Unifying Themes of Bose–Einstein Condensation

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After many years as a phenomenon with only one experimental example, superfluid ^{4}He, the concept of Bose–Einstein condensation (BEC) has in recent years emerged in an array of exciting new experimental and theoretical systems; indeed, the study of BEC has become a field in its own right, no longer primarily a subfield of liquid helium studies. As the articles in this book make apparent, BEC is a common phenomenon occuring in physics on all scales, from condensed matter to nuclear, elementary particle, and astrophysics, with ideas flowing across boundaries between fields. The systems range from gases, liquids, and solids, including semiconductors and metals, to atomic nuclei, elementary particles and matter in neutron stars and supernova explosions. Generally, the bosonic degrees of freedom are composite, originating from underlying fermionic degrees of freedom. Table 1 lists bosonic systems presently under study.

The articles in this book bring out several unifying themes as well as common problems in the study of BEC which transcend specific systems. In this introduction, we give an overview of some of these major themes.

(i) Broken gauge symmetry. What is Bose–Einstein condensation? What is the essential underlying physics? In the past forty years the concept of *broken gauge symmetry* (also described as *off-diagonal long-range order* (ODLRO) or *long-range phase coherence*), which is one of

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Table 1. *Bosons under study*

Particle	Composed of	In	Coherence seen in
Cooper pair	e^-, e^-	metals	superconductivity
Cooper pair	h^+, h^+	copper oxides	high-T_c superconductivity
exciton	e^-, h^+	semiconductors	luminescence and drag-free transport in Cu_2O
biexciton	$2(e^-, h^+)$	semiconductors	luminescence and optical phase coherence in CuCl
positronium	e^-, e^+	crystal vacancies	(proposed)
hydrogen	e^-, p^+	magnetic traps	(in progress)
^{4}He	$^4He^{2+}$, $2e^-$	He-II	superfluidity
^{3}He pairs	$2(^3He^{2+}, 2e^-)$	^{3}He-A,B phases	superfluidity
cesium	$^{133}Cs^{55+}$, $55e^-$	laser traps	(in progress)
interacting bosons	nn or pp	nuclei	excitations
nucleonic pairing	nn or pp	nuclei neutron stars	moments of inertia superfluidity and pulsar glitches
chiral condensates	$\langle \bar{q}q \rangle$	vacuum	elementary particle structure
meson condensates	pion condensate $= \langle \bar{u}d \rangle$, etc. kaon condensate $= \langle \bar{u}s \rangle$	neutron star matter	neutron stars, supernovae (proposed)
Higgs boson	$\langle \bar{t}t \rangle$ condensate (proposed)	vacuum	elementary particle masses

the more remarkable implications of quantum mechanics, has come to the fore as the fundamental principle for understanding BEC. Nozières, Huang, Leggett, Stringari, Stoof, and several other authors in this book discuss the implications of the theory of broken gauge symmetry. In the opening review, Chapter 2, Nozières outlines the essential physics which determines whether a Bose condensate will win over competing phases in a wide variety of systems. Leggett, in the concluding review, Chapter 19, examines the validity of the concept of broken symmetry at a very fundamental level.

The concept of broken gauge symmetry, with the accompanying macroscopic wavefunction describing the condensate, was first introduced in explaining superconductivity and superfluidity. Since then it has also been widely applied in nuclear and elementary particle physics. Bosonic condensates, e.g., with non-vanishing expectation values of quark operators, $\langle \bar{u}u \rangle$, $\langle \bar{d}d \rangle$, $\langle \bar{s}s \rangle$, are believed to be fundamental features of the vacuum and the structure of elementary particles such as the nucleon, underlying the spontaneous breaking of the chiral symmetry of the strong interactions [1]. In fact, the Higgs boson, which gives rise to the breaking of the $SU(2)\times U(1)$ symmetry of the weak interactions, and is invoked to generate the masses of elementary particles, has been suggested as arising from a condensation of the top and antitop quarks, with a non-vanishing expectation value $\langle \bar{t}t \rangle$ in the vacuum [2]. Mesonic condensates, e.g., pion and kaon condensates, which are nuclear analogs of exciton condensates, may be an important feature in dense neutron star matter, as Brown discusses in his article. In nuclear physics, BCS-type pairing is used to explain the reduced moments of inertia of heavier nuclei. Both the neutron and proton components of nuclear and neutron star matter, more generally, undergo pairing to become superfluid.

In high energy physics, the normal state of the vacuum contains a condensate, and thus one can ask the opposite of the question asked in condensed matter physics, namely, can one produce *non*-condensed states? A program to do so, in collisions of ultrarelativistic heavy-ions, has been underway at CERN and Brookhaven. In the normal vacuum, quark condensates, $\langle \bar{q}q \rangle$, are non-zero; in a collision, on timescales of about 10^{-23} s and distance scales of order 10^{-12} cm, it is believed possible to generate regions with $\langle \bar{q}q \rangle = 0$, in which chiral symmetry becomes unbroken and the quarks deconfined [3]. Somewhat later the quarks form hadrons, mostly π mesons, and the state returns to $\langle \bar{q}q \rangle \neq 0$. The dense pion gas might itself Bose condense, although most likely its entropy is too high.

(ii) The telltale signal for BEC. What is the proof that a system is Bose–Einstein condensed? Superfluid helium has been studied for decades, and while we know that it is Bose–Einstein condensed, what is the "smoking gun?" Almost thirty years ago, Hohenberg and Platzman [4] proposed that neutron scattering experiments should yield the classic signal, a delta function for the occupation number of the particles with zero momentum associated with long-range phase coherence. As Sokol discusses in his article in this book, however, such a clean signal is not likely to appear any time soon in the experimental liquid helium data. The reason is that liquid helium is a strongly interacting system, so that the condensate is strongly depleted; furthermore, the short lifetime of the final state of the recoiling atom broadens the neutron scattering data considerably. The neutron scattering data have been shown to be consistent with many-body numerical calculations of helium which predict about 9% of the atoms with zero momentum at $T = 0$ [5, 6]. Nevertheless, the strongest proof of the existence of a condensate in liquid helium probably comes from analysis of the critical exponents of the superfluid phase transition. (They are characteristic of a two-component order parameter.) Obtaining accurate data on critical exponents in other experimental systems may turn out to be much more difficult. In a recent book, Griffin [6] has given a detailed review of the dynamical properties of liquid ^{4}He based on the role of an underlying Bose condensate.

What is the general relation between BEC and superfluidity? Do the superfluid properties of ^{4}He prove that it involves a Bose–Einstein condensate? The classic work of Hohenberg and Martin, as well as Bogoliubov, in the 1960's showed how the two-fluid description of superfluid ^{4}He was a direct consequence of the existence of Bose broken symmetry (see chapter 6 of Ref. [6]). That a condensate is not *necessary* is seen in superfluid two-dimensional Kosterlitz–Thouless systems [7], which, rather than having an infinitely long-ranged phase coherence, have a particle correlation function $\langle \psi^\dagger(r)\psi(0)\rangle$ with a power-law dependence on r. Lasers illustrate that a condensate is not *sufficient*. Huang presents a model of condensation in the presence of a highly disordered substrate to suggest that, more generally, Bose–Einstein condensation is neither necessary nor sufficient for superfluidity of boson systems.

(iii) Bose–Einstein vs. Fermi–Dirac degrees of freedom. Bosonic degrees of freedom are in general composite; indeed the only fundamental bosons in nature appear to be the photon, the three massive vector bosons mediating the weak interaction, the gluon, which mediates the strong interaction, and the graviton. We must thus face the problem of

understanding how bosonic degrees of freedom emerge from underlying fermionic degrees of freedom. Fermionic systems, for example liquid ^{3}He and electrons in metals, become superfluid; to what extent can these systems be regarded as Bose–Einstein condensates? As discussed by Randeria in this book, the same underlying theory based on the formation of Cooper pairs can yield both BEC and BCS in different limits, BEC in the limit of particle–particle correlation length very short compared to the average interparticle spacing, and BCS in the opposite limit. One cannot in general always make a clean distinction between a superfluid system of Cooper pairs and a Bose–Einstein condensate. Superfluidity and superconductivity both stem from the same underlying physics. In either case, the bosonic nature of the pairs is critical for understanding the "super" behavior. Ranninger and others in this volume suggest possible mechanisms for understanding the new cuprate high-T_c superconductors as a BEC of small Cooper pairs.

In condensed-matter physics, the internal fermion degrees of freedom of the nuclei only play the role of determining the statistics of the nucleus, since nuclear energy scales are so vastly larger than atomic scales. The nuclear domain itself provides important examples of the transition between Bose and Fermi degrees of freedom. Examples include the Interacting Boson Model of nuclei, reviewed here by Iachello, the description of mesons in terms of their underlying quark structure by Rho [8] and the deconfinement transition between bosonic mesons and fermionic quarks, as expected in ultrarelativistic heavy-ion collisions and in the early universe [3]. In this book Röpke and Hellmich *et al.* treat the BEC–BCS crossover in nuclear matter, analogous to that discussed by Randeria in superconductors.

(iv) BEC of a weakly interacting gas. In superfluid helium, many features associated with BEC are masked by the strongly interacting character of the liquid [6]. Many of the novel systems currently studied in the laboratory are *weakly interacting* boson gases, whose momentum spectra and other properties can be much more amenable to theoretical analysis. The theory of a weakly interacting gas has been developed at length over the years, starting with London and Bogoliubov, and aspects of the theory are reviewed by Huang, Stoof and Laloë in this volume; Greytak and Silvera review experimental attempts to observe BEC in spin-polarized hydrogen; Castin, Dalibard and Cohen-Tannoudji review the search for BEC of laser-trapped atoms, in particular cesium; and Wolfe, Lin and Snoke as well as Mysyrowicz review experimental work on BEC of excitons, in particular in Cu_2O. Platzman and Mills report on

a proposal for observation of BEC of positronium. All these systems are expected to remain weakly interacting gases at the BEC phase transition, and one expects that their behaviors in the quantum degenerate regime can be well understood [9].

The question of the "smoking gun" for BEC, discussed above, has become more important because of the recent progress on excitons in Cu_2O. Work over the last ten years has shown that excitons can exceed the critical density for condensation, that they exhibit an extremely narrow energy distribution with full width at half maximum much less than $k_B T$, and that at high densities they move without friction through the crystal over macroscopic distances. This system is clearly a highly quantum-degenerate, weakly interacting boson gas. Determining the extent to which this system exhibits off-diagonal long-range order will require ingenuious experimental tests. One has the advantage, compared with ^{4}He, that measurement of exciton recombination luminescence provides fairly direct information about the particle momentum distribution.

An important property of a weakly interacting Bose gas is the phenomenon of "Bose narrowing," i.e., the energy distribution of a weakly interacting gas should decrease in average energy at a given temperature as density increases, opposite to the increase with density in the average energy of a Fermi gas at a given temperature. So far, this effect, which occurs at densities greater than about one-tenth of the critical density for BEC, has been demonstrated only for excitons. Laser Doppler measurements of the energy distribution of cold atoms may see Bose narrowing in the near future, giving the first indication of their quantum degeneracy.

Another prediction for the weakly interacting gas is *spatial* condensation at the center of a three-dimensional potential well. As discussed in the articles by Greytak and by Wolfe *et al.*, this kind of trap is feasible for both atoms and excitons. If BEC occurs in such a trap, the appearance of an extremely narrow peak in the *spatial distribution* of the gas would provide a dramatic "smoking gun."

Finally, weakly interacting gases can allow study of Bose–Einstein statistics far from equilibrium. The process of stimulated emission, familiar for photons, occurs for all scattering processes for bosons, since scattering rates contain a factor $(1 + N_f)$ for each final scattered Bose particle, where N_f is the occupation number of the final state. This enhancement of the density of final states, or *stimulated scattering*, is opposite to the suppression of scattering of fermions by the factor $(1 - N_f)$ from the Pauli exclusion principle; occupied states "attract" other bosons just as fermions "repel" like fermions. This effect can lead to an increase by orders of magnitude of the total scattering rate even in the normal state, which should be observable in time-resolved studies of the momentum spectra or spatial distribution of weakly interacting systems created at or above the critical density but far from thermal equilibrium. The case of short-lived biexcitons in the semiconductor CuCl illustrates this effect. Although the biexciton lifetime is too short for a spontaneous appearance of BEC, when a condensate is placed "by hand" via a laser tuned to the ground state energy or very near to it, the biexcitons at low energy "attract" other biexcitons to the same region of momentum space [10]. Mysyrowicz reviews this work in CuCl here and discusses evidence for the same effect in transport measurements in Cu_2O.

(v) The timescale for formation of BEC. One important feature in all the weakly interacting boson systems discussed here is that the particles have, in general, a *finite lifetime* to remain in the system. Spin-aligned atomic hydrogen can flip its spin (resulting in the formation of a molecule), and atoms can evaporate from a trap. Excitons and positronium are composed of particle–antiparticle pairs and can thus decay; since these are not conserved, what are the experimental consequences of the spontaneously broken gauge symmetry being only approximate? To what extent can one describe systems with finite lifetime as Bose condensed? For example, as discussed in several articles in this book, excitons in Cu_2O appear to move without friction through the crystal when they reach supercritical densities consistent with BEC. Should excitons ever exhibit superfluidity? Kohn and Sherrington, in an oft-cited paper [11], argued that excitons cannot be superfluid in a rigorous sense, because they are particle–antiparticle pairs. Hanamura and Haug [12] responded, however, that Kohn and Sherrington's argument depends on an *equilibrium* picture, while an excitonic condensate is in fact an inherently non-equilibrium ("quasiequilibrium") state. Keldysh briefly addresses this issue in this book. Generally, the problem of seeing evidence of superfluidity on timescales comparable to the particle lifetime is challenging for both theory and experiment. Can one see analogs of persistent currents, or of the Meissner effect in superconductors, or reduced moment of inertia in He-II, and if so, what is the appropriate superfluid mass density?

The controversial question of the timescale for onset of Bose–Einstein condensation is reviewed at length in articles by Kagan and Stoof. The general issue is the growth and evolution in space and time of the boson correlation function, $\langle\psi^\dagger(r)\psi(0)\rangle$. Several timescales are involved: Suppose that a boson gas has density above the critical density for condensation, but is in a metastable state with no condensate present. How long does the quasiparticle distribution of the system take to develop a peak at zero kinetic energy of width less than $k_B T$? How long does it take to develop a *quasicondensate*, i.e., a condensate whose magnitude is locally that of the equilibrium condensate, but with phase fluctuations arising from a random macroscopic distribution of topological defects? Then how long does it take for the condensate to achieve long-range phase coherence, leading to the characteristic delta function at $p = 0$ in the momentum distribution? Finally, when does superfluidity appear?

(vi) Two-dimensional degenerate boson systems. Formation of a quasicondensate is also the basis of the explanation of the Kosterlitz–Thouless (KT) transition of a boson gas in two dimensions, which leads to superfluidity even though a true condensate cannot appear. The KT transition has been seen for (two-dimensional) helium films [7]; Silvera reviews work on observation of a KT transition of spin-polarized hydrogen adsorbed on a surface, and Matsubara *et al.* briefly report on recent experimental work toward this goal. Semiconductor quantum-well structures provide another natural way to examine physics in two dimensions. One such system contains as excitations the "dumbbell exciton" in which the electron and hole in a two-dimensional well are separated by a potential barrier[13] or by an applied electric field [14]. Early indications [14] of a KT transition in this type of structure were later realized to be obscured by trapping in random potentials created by surface irregularities (similar to the scenario considered by Huang in this book) which actually produced a Fermi–Dirac distribution of excitons [15]. The possibility nevertheless still remains for observation of the Kosterlitz–Thouless transition with better-grown quantum-well structures.

Various kinds of two-dimensional semiconductor structures have been proposed which contain charged bosonic excitations that could in principle become superfluid, conceivably at room temperature, and which would be superconductors. "Magnetoexcitons," which have received much attention in recent years [16], move perpendicular to applied magnetic and electric fields in a two-dimensional well, and could undergo Bose–Einstein condensation [17, 18]. In this volume Shvartsman and Golub speculate that the possibility exists for engineering the band structure of GaAs to yield a "bihole," a boson in two dimensions with charge $+2e$. Since band-structure engineering of semiconductor structures has reached such an advanced state, one can imagine great advances in the near future in the manufacture of new kinds of superconductors based on BEC.

(vii) BEC and lasing. While excitons in Cu_2O have lifetimes long compared to interaction times, in most semiconductors the interaction time is comparable to the lifetime of the excitons due to coupling to the photon field. In the limit of very short exciton lifetime, the excitonic BEC essentially becomes indistinguishable from a laser. The observation of a narrow photon emission peak from a semiconductor does not therefore immediately indicate the presence of an excitonic BEC, since in many cases it may equally well be described as superradiant emission due to lasing [12]. As discussed by Keldysh in this volume, lasing and excitonic (or biexcitonic) BEC can be seen as two limits of the same theory. Lasing occurs in the case of strong electron–photon coupling (recombination rate large compared to the interparticle scattering rate) while the excitonic condensate occurs in the case of weak electron–photon coupling (recombination rate slow compared to interparticle scattering rate). A laser, in fact, can be seen as a Bose–Einstein condensate in which the long-range order involves the photon states, while in the excitonic condensate the long-range order involves the electronic states [19]. Many of the same issues of the timescale for onset of condensation, discussed above, have already been discussed in the context of the onset of lasing [20]. The polariton condensate, reviewed by Keldysh (see also Beloussov and Shvera) in this volume, represents a middle case between lasing and excitonic BEC, since the polariton has a mixed character with both exciton-like and photon-like behavior.

The overlap between BEC and lasing has been illustrated recently in the context of understanding the origin of dark matter in the early universe, where it has been argued that decay of a heavy species of neutrino into a light fermion and a boson could give rise either to a Bose–Einstein condensate [21] or to a "neutrino laser" [22].

(viii) BEC of small clusters of particles. The usual theory of BEC assumes the thermodynamic limit of an infinite system. The concept of bosonization, including BEC and superfluidity, has become valuable in understanding the properties of nuclei, however, in which pairs of nucleons bind via the strong force to produce effective bosonic degrees of freedom. Iachello reviews the theory of bosonization; Röpke as well as Hellmich *et al.* discuss the crossover from BEC to BCS in the theory of nuclear matter. These concepts work quite well in small nuclei, which are systems far from the thermodynamic limit; this is perhaps not so surprising since numerical models of helium using of order 64 atoms have been shown to reproduce the bulk superfluid properties of helium [23]. The theory of BEC of small clusters of bosons [24] may find extension in

the future to atomic and molecular systems, since the experimental study of atomic clusters has progressed tremendously in recent years. What is the minimum size of a system that we may still call a Bose–Einstein condensate?

(ix) BEC in random potentials. The appearance of a Bose condensate in disordered systems is a very active area of research in condensed-matter physics (the so-called "dirty boson" problem.) The key issues are highlighted in the review article by Nozières, and some specific aspects are treated in the articles by Huang and Stringari, but this important topic is not reviewed in depth in this book. Experimental systems to which this theory may apply include excitons in quantum wells with rough surfaces, and helium in porous media such as Vycor, the superconductor–insulator transition in disordered films, and vortices in type-II superconductors.

As one can see from this short survey, the abundant themes in BEC go beyond specific system properties, and present a unifying basis to the study of the phenomenon in different systems. We hope that this volume will serve as a stimulus to deeper understanding of these issues.

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References

[1] J. Gasser and H. Leutwyler, Phys. Reports **87**, 77 (1982).
[2] C. T. Hill, Phys. Lett. B **266**, 419 (1991) and references therein.
[3] See, e.g., *International conferences on quark matter and ultra-relativistic nucleus–nucleus collisions*, Nucl. Phys. A **544** (1992); in press (1994).
[4] P. C. Hohenberg and P. M. Platzman, Phys. Rev. **152**, 198 (1966).
[5] D.M. Ceperley and E.L. Pollock, Phys. Rev. Lett. **56**, 351 (1986).
[6] A. Griffin, *Excitations in a Bose-Condensed Liquid* (Cambridge University Press, Cambridge, 1993).
[7] For a review, see D.R. Nelson, in *Phase Transitions and Critical Phenomena* **7**, C. Domb and J.L. Lebowitz, eds. (Academic, New York, 1983).
[8] M. Rho, "Cheshire Cat Hadrons," Phys. Reports, in press (1994).
[9] See, e.g., V.N. Popov, *Functional Integrals and Collective Excitations*, (Cambridge University Press, Cambridge, 1987), ch. 6.

[10] N. Peghambarian, L.L. Chase, and A. Mysyrowicz, Phys. Rev. B **27**, 2325 (1983).
[11] W. Kohn and D. Sherrington, Rev. Mod. Phys. **42**, 1 (1970).
[12] E. Hanamura and H. Haug, Sol. State Comm. **15**, 1567 (1974); Phys. Reports **33**, 209 (1977).
[13] F.M. Peeters and J.E. Golub, Phys. Rev. B **43**, 5159 (1991).
[14] T. Fukuzawa *et al.*, Phys. Rev. Lett. **64**, 3066 (1990).
[15] J.A. Kash *et al.*, Phys. Rev. Lett. **66**, 2247 (1991).
[16] D.S. Chemla J.B. Stark, and W.H. Knox, *Ultrafast Phenomena VIII*, (Springer, Berlin, 1993), p. 21.
[17] I.V. Lerner and Yu. E. Lozovik, Sov. Phys. JETP **53**, 763 (1981).
[18] D. Paquet *et al.*, Phys. Rev. B **32**, 5208 (1985).
[19] The analogy between a laser and superfluid ^{4}He is nicely reviewed by P.C. Martin, in *Low Temperature Physics— LT9*, J.E. Daunt, D.O. Edwards, F.J. Milford, and M. Yaqub, eds. (Plenum, New York, 1965), p. 9.
[20] See, e.g., H. Haken, Rev. Mod. Phys. **47**, 67 (1975).
[21] J. Madsen, Phys. Rev. Lett. **69**, 571 (1992).
[22] N. Kaiser, R. A. Maleney, and G. D. Starkman, Phys. Rev. Letters **71**, 1128 (1993).
[23] Finite size effects are discussed by E.L. Pollock, Phys. Rev. B **46**, 3535 (1992).
[24] Ph. Sindzingre, M. Klein and D. M. Ceperley, Phys. Rev. Lett. **63**, 1601 (1989).