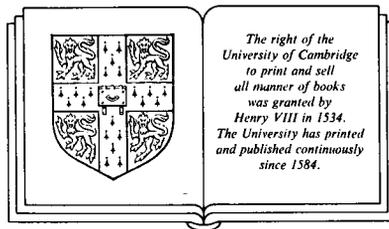


Recombination in semiconductors

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Introduction

A systematic interest in recombination in semiconductors dates roughly from 1950 and gave rise for example to the Shockley–Read–Hall statistics in 1952 and the application of detailed balance to radiative processes in semiconductors in 1954. But our story really started with quantum mechanics and its application to solids. In contrast to its junior cousin, the black hole, the hole of semiconductor physics was first seen experimentally (in the anomalous Hall effect) and quantum mechanics was used to elucidate it [1]. Quantum mechanics was also used later to propose the band model of a semiconductor [2]. Actually, the copper oxide plate rectifier had been a useful solid-state device since the early days of quantum mechanics in the 1920s, but its action came to be understood only just before the war using electrons, holes and the band model [3]. This work has been reviewed for example by Mott and Gurney [4] and by Henisch [5]. Solid-state electronics was already in the air then, and rudimentary solid-state amplification had been proposed by Lilienfeld [6] in the late 1920s and established by Hilsch and Pohl [7] in 1938, using potassium bromide crystals. A useful semiconductor ‘triode’ was clearly ready to be born in 1938/9. But the war intervened. Still, solid-state detection was now important for radar, and programs to study silicon and germanium were initiated partly with government funding, notably at Purdue University under K. Lark-Horovitz. At the end of the war one could claim that what Shockley called ‘one-current theories’ of metal-semiconductor rectification [8] were in reasonable shape, apart from a little tinkering here and there, for example, by the present author (see [5]). Of course, the study of surfaces was then, and continues to be, a very active area of research.

The two-current theories, and with them recombination, came a few years after the war when in 1947 Bardeen and Brattain discovered the point contact transistor along with minority carrier injection [9]. Electron–hole pair generation was also

studied and was rather like another cousin, this time a senior one, namely electron–positron pair creation. (This requires energy $2m_0c$ which corresponds in the solid state to $E_G = 2E_0 = 2m_0^*v^2$, where E_G is the energy gap, E_0 is the nonparabolicity parameter (1.6.16) and v is a saturation velocity (1.6.31) which indeed corresponds to c .) The junction transistor followed, and was discussed by Shockley in a post-deadline paper at an international conference held in Reading, England, in July 1950 [10]. (It did not appear in the proceedings [11].)

Other developments followed, as is well known: the onward march of silicon, many new devices, including solar cells which enabled space exploration to proceed, GaAs and other III–V compounds, integrated circuits, microminiaturization, ever faster and more compact computers, etc. With vacuum microelectronics the wheel may come full circle. The story is well outlined elsewhere ([12]–[14]).

If we fasten our attention on the year 1950 for the systematic beginning of our subject, we can think of the Reading Conference of July 1950 as the appropriate event. Indeed, seven of the people mentioned above were present, as is clear from the group photograph given as the Frontispiece. (Additions and corrections to the identifications are needed and will be welcomed.) It is a historical document, particularly since the Reading Conference came to be regarded as the first of a series of international conferences on the physics of semiconductors which runs as follows: (2) Amsterdam, 1954; (3) Garmisch, 1956; (4) Rochester, 1958; (5) Prague, 1960; (6) Exeter, 1962; (7) Paris, 1964; (8) Kyoto, 1966; (9) Moscow, 1968; (10) Cambridge USA, 1970; (11) Warsaw, 1972; (12) Stuttgart, 1974; (13) Rome, 1976; (14) Edinburgh, 1978; (15) Kyoto, 1980; (16) Montpellier, 1982; (17) San Francisco, 1984; (18) Stockholm, 1986; (19) Warsaw, 1988; (20) Thessaloniki, 1990. If someone exists who attended them all, he should be given a medal for devotion to the subject, longevity, and willingness to travel.

This book, then, is devoted to the main aspects of the physics of recombination in semiconductors, omitting related topics that are well covered elsewhere, such as band theory, resonances, details of phonon effects, and amorphous systems. The concepts are introduced so that graduate students who are beginning research can follow the argument. Some things may *look* complicated, but they *are* explained, so that some of the work (chapter 1, sections 2.1 to 2.3, and the beginnings of chapters 3 and 4) can be studied already in an undergraduate course. My idea was to make the book almost completely self-contained with an emphasis on general principles. This should enable a reader to make new applications while it will also detach the book a little from the precise contemporary state of research, so that, with luck, it will not go out of date too quickly. In a first study of the book the portions in small print can be omitted. A short guide to the book is given at the end of this Introduction.

It will be seen that an attempt to cover the main topics in the recombination area has been made. However, not all relevant subjects could be discussed, and I regret the disappointment this may cause some readers. In general, these omissions are due to my lack of competence in these areas, coupled with the need to keep the book from becoming too long. The knowledgeable reader will, perhaps, be recompensed by finding relatively new material already early in the book and in unexpected places.

Imbedded in chapters 1 and 2 is the material for a possible slim volume called *The Partition Function Approach to Semiconductor Kinetics*. For, on looking through this book, it became clear to me that this rather useful tool, largely developed and used by my collaborators and myself over the last decades, is not as widely known as it should be. Perhaps it has not been popularized with sufficient skill. Hence, the possibility of the above mentioned slim volume. This is no 'plug' for an (unwritten) book. On the contrary, the careful reader of the present book will already know all there is to be taught by me, and therefore will not need to have it, should it ever materialize.

I have tried to give an indication of current thought with regard to most topics, but at the same time I wanted to preserve a sense of the historical development of our subject. For example the paper [1.8.10] by Debye and Conwell (1954) is not often cited now, but it was very influential at the time. I have also cited papers which, while good and relevant, have had a rather low citation count, thus helping to 'save' them for possible future work – [5.2.57] is an example. Thus many papers are cited. This is done by number (for example [1.8.10] is reference 10 in section 1.8), for which I apologize, but it has saved much space, as many papers have three authors and some have more, and it has helped to keep the continuity of argument.

For help with the identification of the persons in the frontispiece I thank Dr P.C. Banbury (Reading), Dr R.N. Bloomer (Locksheath, Southampton), Prof. G. Busch (Zürich), Prof. H.K. Henisch (State College, Pa.), Sir George MacFarlane (Esher, UK), Dr T.S. Moss (Malvern, UK), and Dr A.J. Vink (V.D. Waalre, The Netherlands).

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* Now GEC-Marconi Materials Technology Ltd.

A short guide to the contents of this book

References are to sections unless stated otherwise.

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	Statistics for combining recombination channels: 2.2.4, 2.3.7		
	Surface recombination and grain boundary barrier heights: 2.7		
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			Tunneling: 2.5.5, 2.5.6
Mixture of above	Recombination in low-dimensional structures (chapter 7)		

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It is common for an author to thank his family for suffering his absences during the writing of his masterpiece. The present case is slightly different. It is my family who are thanking me for keeping away from under their feet by undertaking the task, the result of which I have now pleasure to present.