

# *Introduction to Structural Dynamics and Aeroelasticity*

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**CAMBRIDGE**  
**UNIVERSITY PRESS**

PUBLISHED BY THE PRESS SYNDICATE OF THE UNIVERSITY OF CAMBRIDGE  
The Pitt Building, Trumpington Street, Cambridge, United Kingdom

CAMBRIDGE UNIVERSITY PRESS  
The Edinburgh Building, Cambridge CB2 2RU, UK  
40 West 20th Street, New York, NY 10011-4211, USA  
477 Williamstown Road, Port Melbourne, VIC 3207, Australia  
Ruiz de Alarcón 13, 28014 Madrid, Spain  
Dock House, The Waterfront, Cape Town 8001, South Africa  
<http://www.cambridge.org>

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First published 2002

Printed in the United States of America

*Typeface* Times New Roman 10/12 pt.    *System* L<sup>A</sup>T<sub>E</sub>X 2<sub>ε</sub> [TB]

*A catalog record for this book is available from the British Library.*

*Library of Congress Cataloging in Publication Data*

Hodges, Dewey H.

Introduction to structural dynamics and aeroelasticity / Dewey H. Hodges, G. Alvin Pierce.  
p. cm. – (Cambridge aerospace series ; 15)

Includes bibliographical references and index.

ISBN 0-521-80698-4

1. Airframes – Design and construction. 2. Structural design. 3. Space  
vehicles – Dynamics. 4. Aeroelasticity. I. Pierce, G. Alvin. II. Title. III. Series.

TL671.6 .H565 2002

629.134'31 – dc21

2001052552

ISBN 0 521 80698 4 hardback

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## *Introduction*

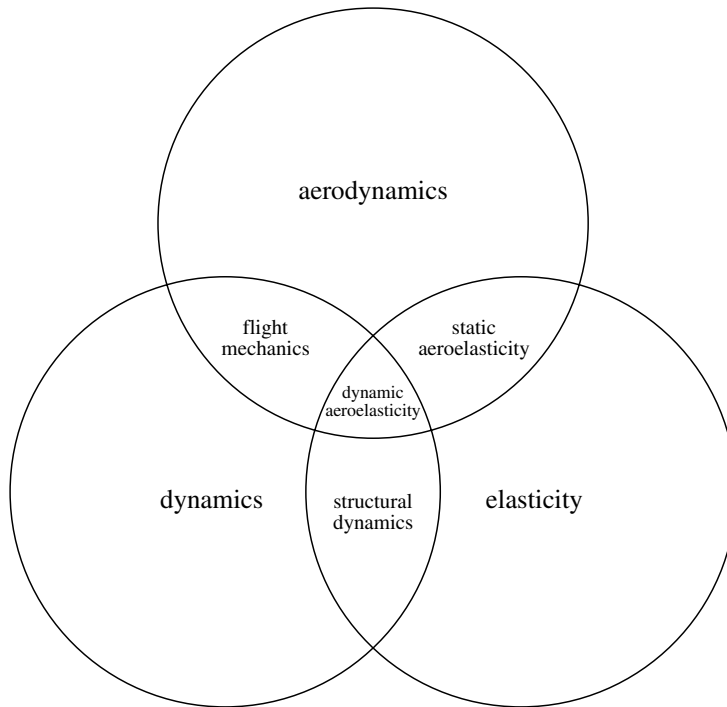
Aeroelasticity is the term used to denote the field of study concerned with the interaction between the deformation of an elastic structure in an airstream and the resulting aerodynamic force. The interdisciplinary nature of the field can be best illustrated by Fig. 1.1, which depicts the interaction of the three disciplines of aerodynamics, dynamics, and elasticity. Classical aerodynamic theories provide a prediction of the forces acting on a body of a given shape. Elasticity provides a prediction of the shape of an elastic body under a given load. Dynamics introduces the effects of inertial forces. With the knowledge of elementary aerodynamics, dynamics, and elasticity, the student is in a position to look at problems in which two or more of these phenomena interact. One of those areas of interaction is the field of flight mechanics, which most students have studied in a separate course by the senior year. The present text will consider the three remaining areas of interaction:

- between elasticity and dynamics (structural dynamics),
- between aerodynamics and elasticity (static aeroelasticity), and
- among all three (dynamic aeroelasticity).

Because of their importance to aerospace system design, these are appropriate for study in an undergraduate aeronautics/aeronautical engineering curriculum. In aeroelasticity one finds that the loads depend on the deformation (aerodynamics), and the deformation depends on the loads (structural mechanics/dynamics); thus one has a coupled problem. Consequently, prior study of all three constituent disciplines is necessary before a study in aeroelasticity can be undertaken. Moreover, a study in structural dynamics can be helpful to develop concepts that are useful in solving aeroelasticity problems, such as the modal representation.

It is of interest to note that aeroelastic phenomena have played a major role throughout the history of powered flight. The Wright brothers utilized controlled warping of the wings on their Wright Flyer in 1903 to achieve lateral control. This was essential to their success in achieving powered flight because the aircraft was laterally unstable owing to the significant anhedral of the wings. Earlier in 1903 Samuel Langley made two attempts to achieve powered flight from the top of a houseboat on the Potomac River. His efforts resulted in catastrophic failure of the wings due to aeroelastic divergence caused by insufficient torsional stiffness. Torsional divergence phenomena were a major factor in the predominance of the biplane design until the early 1930s when “stressed skin” metallic structural configurations were introduced to provide adequate torsional stiffness for monoplanes.

The first recorded and documented case of flutter in an aircraft occurred in 1916. The Handley Page O/400 bomber experienced violent tail oscillations as the result of the lack of a torsion rod connection between the port and starboard elevators, an absolute design requirement of today. The incident involved a dynamic twisting of the fuselage to as much as 45 degrees in conjunction with an antisymmetric flapping of the elevators. Catastrophic failures due to aircraft flutter became a major design concern during the First World War and remain so today. R. A. Frazer and W. J. Duncan at the National Physical Laboratory in England compiled a classic document on this subject entitled “The Flutter of Aeroplane



**Figure 1.1** Schematic of the field of aeroelasticity.

Wings” as R&M 1155 in August 1928. This small document (about 200 pages) became known as “The Flutter Bible.” Their treatment for the analysis and prevention of the flutter problem laid the groundwork for the techniques in use today.

Another major aircraft design concern that may be classified as a static aeroelastic phenomenon was experienced in 1927 by the Bristol Bagshot, a twin-engine, high-aspect-ratio English aircraft. As the speed was increased the aileron effectiveness decreased to zero and then became negative. This loss and reversal of aileron control is commonly known today as aileron reversal. The incident was successfully analyzed and design criteria were developed for its prevention by Roxbee Cox and Pugsley at the Royal Aircraft Establishment in the early 1930s. Although aileron reversal does not generally lead to a catastrophic failure, it can be quite dangerous and is thus an essential design concern. It is of interest to note that during this period of the early 1930s it was Roxbee Cox and Pugsley who proposed the name “aeroelasticity” to describe these phenomena, which are the subject of this text.

In the design of aerospace vehicles, aeroelastic phenomena can result in a full spectrum of behavior from the near benign to the catastrophic. At the near benign end of the spectrum one finds passenger and pilot discomfort. One moves from there to steady-state and transient vibrations that slowly cause the aircraft structure to suffer fatigue damage at the microscopic level. At the catastrophic end, there are aeroelastic instabilities that can quickly destroy an aircraft and result in loss of human life without warning. Aeroelastic problems that need to be addressed by the aerospace system designer can be mainly static in nature, meaning that inertial forces do not play a significant role, or they can be strongly influenced by inertial forces. Although not the case in general, the analysis of some aeroelastic phenomena can be undertaken by means of small deformation theories. Aeroelastic phenomena may

strongly affect the performance of an aircraft, positively or negatively. They may also determine whether its control surfaces perform their intended functions well, poorly, or even in the exact opposite manner of that which they are intended to do. It is clear then that all these studies have very important practical consequences in many areas of aerospace technology. The design of modern aircraft and space vehicles is characterized by the demand for extremely lightweight structures. Therefore, the solution of many aeroelastic problems is a basic requirement for achieving an operationally reliable and structurally optimal system. Aeroelastic phenomena also play an important role in turbomachinery, in wind energy converters, and even in the sound generation of musical instruments.

The most commonly posed problems for the aeroelastician are stability problems. Although the elastic moduli of a given structural member are independent of the speed of the aircraft, the aerodynamic forces strongly depend on it. It is therefore not difficult to imagine scenarios in which the aerodynamic forces “overpower” the elastic restoring forces. When this occurs in such a way that inertial forces have little effect, we refer to this as a static aeroelastic instability – or “divergence.” In contrast, when the inertial forces are important, the resulting dynamic instability is called “flutter.” Both divergence and flutter can be catastrophic, leading to sudden destruction of the vehicle. Thus, it is vital for aircraft designers to know how to design lifting surfaces that are free of such problems. Most of the treatment of aeroelasticity in this text is concerned with stability problems.

Much of the rest of the field of aeroelasticity involves a study of the response of aircraft in flight. Static aeroelastic response problems constitute a special case in which inertial forces do not contribute and in which one may need to predict the lift developed by an aircraft of given configuration at a specified angle of attack, or determine the maximum load factor such an aircraft can sustain. Also, problems of control effectiveness and aileron reversal fall under this category. When inertial forces are important, one may need to know how the aircraft reacts in turbulence or in gusts. Another important phenomenon is buffeting, which is characterized by transient vibration induced by wakes behind wings, nacelles, or other components of the aircraft.

All the above are treatable within the context of a linear analysis. Mathematically, linear aeroelastic response and stability problems are complementary. That is, instabilities are predictable from examining the situations under which homogeneous equations possess nontrivial solutions. Response problems, however, are generally based on solution of nonhomogeneous equations. When the system goes unstable, a solution to the nonhomogeneous equations ceases to exist, while the homogeneous equations associated with a stable configuration have no nontrivial solution.

Unlike the predictions from linear analyses, in real aircraft it is possible for self-excited oscillations to develop, even at speeds less than the flutter speed. Moreover, large disturbances can bump a system that is predicted to be stable by linear analyses into a state of large oscillatory motion. Both situations can lead to steady-state periodic oscillations for the entire system called limit-cycle oscillations (LCO). In such situations, there can be fatigue problems leading to concerns about the life of certain components of the aircraft as well as passenger comfort and pilot endurance. To capture such behavior in an analysis, the aircraft must be treated as a nonlinear system. Although of great practical importance, nonlinear analyses are beyond the scope of this textbook.

The organization of the text is now presented. To describe the dynamic behavior of conventional aircraft, the topic of *structural dynamics* is introduced in Chapter 2. This is the study of dynamic properties of continuous elastic configurations, which provides a means of analytically representing a flight vehicle’s deformed shape at any instant of time. We begin

with very simple systems, such as vibrating strings, and move up in complexity to beams in torsion and finally to beams in bending. The introduction of the modal representation and its subsequent use in solving aeroelastic problems is the paramount emphasis of this chapter. A very brief introduction to the methods of Ritz and Galerkin is also included.

Chapter 3 treats *static aeroelasticity*. Therein we concern ourselves with static instabilities, steady airloads, and control effectiveness problems. Again, we begin with simple systems, such as elastically restrained rigid wings. We move up to wings in torsion and finish the chapter with a treatment of swept wings undergoing elastically coupled bending and torsion deformation. Finally, Chapter 4 treats *aeroelastic flutter*, which is associated with dynamic aeroelastic instabilities due to the mutual interaction of aerodynamic, elastic and inertial forces. A generic lifting surface analysis is first presented, and this is followed by illustrative treatments involving simple “typical section models.” Engineering solution methods for flutter are discussed, followed by a brief presentation of unsteady aerodynamic theories, both classical and modern. The chapter culminates with an application of the modal representation to the flutter analysis of flexible wings and a discussion of the flutter boundary characteristics of conventional aircraft. It is important to note that central to our study in these last two chapters are the phenomena of divergence and flutter, which typically result in catastrophic failure of the lifting surface and may lead to subsequent destruction of the flight vehicle.

Lists of references for structural dynamics and aeroelasticity are included, along with an appendix in which Lagrange’s equations are derived and illustrated.