THE PHYSICS AND
MATHEMATICS OF
ADIABATIC SHEAR BANDS

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Introduction: Qualitative description and one-dimensional experiments

Through words and pictures the first half of this chapter defines what the phrase “adiabatic shear band” means. It gives a qualitative description of the appearance and occurrence of adiabatic shear bands, and by simple inference it also gives some rudimentary idea of the typical mechanical and thermal history that accompanies their formation. A discussion of their importance as damage sites and a qualitative description of the basic mechanics that drives them follows. Then, after a brief review of earlier work including some recent surveys and symposiums, Section 1.1 closes with a short discussion of the implications that adiabatic shear bands have for the predictive accuracy of numerical analysis and computational design.

The second half of the chapter gives an overview of the techniques and typical results that experimentalists have developed in trying to gain a more quantitative picture of the phenomenon. For the most part, these techniques have involved ingenious use of the well-known split Hopkinson pressure bar (or Kolsky bar), but a limited amount of work has also been done by other means. There are really two parts to this work. First it is necessary to explore the fundamental constitutive behavior of a given material at high strain rates and various temperatures. Only then does it make sense to measure the forces, temperatures, strains, and so on that develop during the formation of an adiabatic shear band. This is true because the formation of a shear band is merely the consequence of locally evolving material response to a particular, externally imposed environment after homogeneous deformation has become unstable.

There is a theoretical counterpart to this experimental approach. The fundamentals of large plastic deformation, including a reasonably coherent constitutive theory, ought to be understood before the concrete initial-boundary value problems of shear localization can be most fruitfully explored. The remainder of the book is organized with this last thought in mind.
1. Introduction

1.1 Qualitative Features of Adiabatic Shear Bands

1.1.1 Basic Morphology

An adiabatic shear band is a narrow, nearly planar or two-dimensional region of very large shearing that sometimes occurs in metals and alloys as they experience intense dynamic loading. Once the band is fully formed, the two sides of the region are displaced relative to each other, much like a mode II or mode III crack, but the material still retains full physical continuity from one side to the other. The thickness of the most heavily sheared region may be of the order of only a few tens of micrometers or even less, whereas the lateral extent may be many millimeters or even centimeters in length. Thus the aspect ratio of the sheared region may be in the hundreds or even thousands. Figure 1.1 shows a typical example where a band has been driven into a 1018 cold-rolled steel specimen by a flat-ended projectile. The picture shows a cross section taken perpendicular to both the bottom and vertical side of the cylindrical crater. Note the long, narrow appearance of the band, beginning at a corner of the crater where a stress concentration was introduced by the impact, and extending deep into the specimen in the direction of impact. Figure 1.2 shows a band, also introduced by impact of a projectile, in a 2014-T6 Al alloy. At the scales shown the internal structure of the fully formed bands themselves cannot be seen in either micrograph. In both cases, however, the intense shear in the surrounding material can be seen, achieving shear strains much greater than one in both

![Figure 1.1. Adiabatic shear band in 1018 cold-rolled steel. Band formed at the corner of an impact crater. Impact speed, 100 m/s. (Reproduced from Rogers 1983, with permission from Kluwer Academic/Plenum Publishers.)](image-url)
1.1 Qualitative features of adiabatic shear bands

Figure 1.2. Adiabatic shear band in 2014-T6 Al alloy. Band formed at the corner of an impact crater. (Reproduced from Wingrove 1973. Copyright 1973 by the Minerals, Metals and Materials Society and ASM International.)

cases (as estimated by the tangent of the flow lines). In addition, Figure 1.2 gives the impression of very large deformation within the band. A comparison of the relative positions of the dark zones on either side of the band, as indicated by the letters A and B, suggests a relative displacement of perhaps twenty to thirty times the apparent width of the band itself. Accordingly, the maximum shear strain within the band must be substantially greater than ten by even the crudest estimate. Figure 1.3 shows a closer view of an adiabatic shear band that was produced by a punch driven explosively through a rolled, high strength, Ni-Cr steel alloy. The lines that are visible in the photomicrograph are chemical inhomogeneities that lay approximately in the rolling planes, which originally were horizontal in the picture. In this example, the shear strain exterior to the band proper is approximately one, and again, by direct examination of the slope of the marker lines, the strain within the band is greater than ten.

These three examples have introduced several of the principal characteristics of adiabatic shear bands: thinness perpendicular to the direction of maximum shearing, high aspect ratio, high external strain, extreme internal strain, and dynamic generation, the latter inferred from projectile impact and punching, and in turn implying generation under high strain rates. In each of the three cases, the band cuts straight through the material, apparently without regard to the microstructure. This observation seems to hold true in the vast majority of cases, but branching is sometimes observed also. In addition, we observe that bands can form in different materials and that their appearance may be different in different materials.

This last point has been noted in the older literature by a referral to deformed and transformed bands. Transformed bands, like the two steels, show white after etching in preparation for photomicroscopy, whereas deformed bands, like the aluminum alloy, do not. It has often been speculated that a phase transformation
1. Introduction

Figure 1.3. Adiabatic shear band in a Ni-Cr steel alloy. Band formed from a punch driven explosively through the specimen. White lines are bands of chemical inhomogeneity and were horizontal before punching. (Reproduced from Moss 1981, with permission from Kluwer Academic/Plenum Publishers.)

has occurred in a transformed band, but at least in some cases this does not appear to be so. Beatty, Meyer, Meyers, and Nemat-Nasser (1992) showed that the center of a shear band in a 4340 steel (RC52 hardness before shearing) contained very fine grains (8–20 nm), probably as a result of grain refinement caused by the severe deformation, but no indication of a transformation from martensite to austenite was apparent. Meunier, Roux, and Moureaud (1992) obtained similar results for an armor steel, as did Chichili (1997) for alpha Ti. In any event, when the internal structure of the band is visible, as in Figure 1.3, the lines of deformation appear to extend more or less smoothly and continuously.
1.1 Qualitative features of adiabatic shear bands

Figure 1.4. Tip of an adiabatic shear band showing the transition from intense localization to a more diffuse deformation. (Reproduced from Meyers and Wittman 1990. Copyright 1990 by the Minerals, Metals and Materials Society and ASM International.)

through the white etching region, suggesting that even if a phase transformation has occurred, the kinetics are such that it has not affected the basic processes of deformation.

After deformation has stopped, however, Figure 1.4 indicates that the end or tip of a shear band simply becomes diffuse as the intense deformation within the band transitions into the general deformation of surrounding material. In other cases, as in Figure 1.5, there is no obvious deformation pattern through the band. Here there is a rough appearance, as if many smaller grains had been produced by the thermomechanical deformation or perhaps by recovery processes. Again the kinetics and timing of the transformation are uncertain.

Figure 1.6 shows a typical microhardness profile across a shear band. Usually the center of the band shows greatly increased hardness, as in this case,

Figure 1.5. Adiabatic shear band in Ti. Material appears to have recrystallized after band formation. (Reproduced from Grebe, Pak, and Meyers 1985. Copyright by the Minerals, Metals and Materials Society and ASM International.)
which might occur from a cycle of work hardening and heating, followed by rapid quenching. Occasionally the profile shows several peaks in hardness, as in Figure 1.7. Besides the morphological characteristics listed in the last paragraph, then there also are indications that somewhat elevated temperatures have occurred within the fully formed band.
1.1 Qualitative features of adiabatic shear bands

1.1.2 Occurrence

Adiabatic shear bands have been observed to form in many circumstances. Perhaps the most well-known example is ballistic impact, in which adiabatic shear has long been observed in targets and more recently in penetrators as well. Timothy and Hutchings (1985) reported shear bands that formed in the bottom of a crater formed by the impact of a hard steel ball on a target material of Ti-6Al-4V (see Figure 1.8). This study shows that shear bands can form in regions that are free from obvious stress concentrations, such as those examined by Rogers (1979) in his studies. Traditionally shear bands have been regarded as deleterious to the performance of either penetrators or armors, as in projectile shatter or target plugging. More recently, however, Magness (1994) proposed that adiabatic shear can be a beneficial mechanism that enhances the superior performance of U3/4Ti as a penetrator material, even beyond that expected from its high density. The idea is that adiabatic shear failure, occurring at an optimum strain, allows the penetrator material to cease plastic flow and, by failing, to be able to exit the zone of interaction with the target at a time earlier than would have been possible by plastic flow alone. The net result is that the cavity forced into the target is narrower and deeper than would otherwise have been the case. Figure 1.9 is a sketch of the proposed mechanism, which is now widely accepted as the correct explanation. Figure 1.10 shows the characteristic shape of a penetrator in free flight after perforating a target in which adiabatic shear has allowed the penetrator to be “self-sharpening.”

Targets sometimes show unusual shear failures. Figure 1.11 shows an example in which the target material tends to fail on surfaces more or less parallel...
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Figure 1.9. Sketch of flow and failure by adiabatic shear in the tip of a penetrator as it penetrates a target. (Mechanism proposed by Magness 1993; reproduced with permission from Metal Powder Industries Federation/APMI International.)

to the path of penetration. Notice the roughness of the failure zone where adiabatic shear has apparently allowed target material to break away intermittently as the penetrator advanced. In this case the total cavity is much wider than would be expected from gross plastic flow alone. In fact, shear failure seems to occur near surfaces of maximum strain rate, as calculated by Batra and Wright (1986). Figure 1.12 shows an example in which shear zones have formed ahead of the advancing penetrator, much like bow waves, but because the shearing

Figure 1.10. Flash x-ray of a U-8Mo penetrator after perforation of a steel target. The chiseled nose is evidence that adiabatic shear has played a strong role in eroding the penetrator. (Reproduced from Magness 1994, with permission from Elsevier Science.)
1.1 Qualitative features of adiabatic shear bands

Figure 1.11. Tunnels formed in targets made of Ti-6Al-4V after being struck by a small rod made of 90W-7Ni-3Fe at 1450 m/s. Top figure shows incipient shear parallel to the tunnel at a distance that corresponds to the maximum gradient in the velocity field. Bottom figure shows fully developed shear. (G. E. Hauver, unpublished photographs, with permission from the Terminal Effects Division of the Army Research Laboratory.)

Figure 1.12. Tunnel formed in a target made of S7 tool steel at RC30 after being struck by a small rod made of 90W-7Ni-3Fe at 1450 m/s. Figure shows a sequence of sheared regions that form intermittently ahead of the penetrator, like bow waves. (G. E. Hauver, unpublished photographs, with permission from the Terminal Effects Division of the Army Research Laboratory.)

takes place on material surfaces, they cannot advance with the penetrator and so are perforated as penetration continues. New surfaces form and are perforated in sequence until finally penetration stops.

There are also many industrial processes in which adiabatic shear bands may form (Semiatin, Lahoti, and Oh 1983). Figure 1.13 shows a segmented chip formed during machining of Ti-6Al-4V, where the segments are separated by adiabatic shear bands. The formation of such chips places a limit on the quality of surface finish that can be achieved. Additionally, the figure shows...
that shear bands can form and reform intermittently if the loading conditions are maintained, as also seems to be the case in Figures 1.9, 1.11, and 1.12. Forging or upsetting can also produce unwanted adiabatic shear bands. Figure 1.14 shows a cross-shaped region in the middle of the specimen caused by side pressing (Semiatin et al. 1983). A high rate compression has been shown to produce adiabatic shear bands in some materials. Wulf (1978) found shear bands in an Al alloy after compression testing in a split Hopkinson pressure bar; see Figure 1.15. Chen and Meyers (personal communication, but see also Chen, Meyers, and Nesterenko 1999 for further details) found that they could produce shear bands in Ta with high rate compression at 77 K, Figure 1.16, but not at room temperature. Grady, Asay, Rohde, and Wise (1983) observed a cross-hatched deformation pattern in Al after shock impact (Figure 1.17), which they interpreted as possibly being adiabatic shear bands. These examples
1.1 Qualitative features of adiabatic shear bands

Figure 1.15. Adiabatic shear band formed in a compression specimen of 7039 Al at a strain rate of $8 \times 10^3$ s$^{-1}$. Total strain is 0.26. (Reproduced from Wulf 1978, with permission from Elsevier Science.)

demonstrate that adiabatic shear bands may form in many materials during many kinds of deformation.

1.1.3 Importance

Their mere existence would not make adiabatic shear bands worthy candidates for extensive study if they did not significantly alter the subsequent behavior and performance of the material in question. However, as suggested by the ballistic examples, they can have a very substantial effect. Adiabatic shear bands commonly act as sites for further damage and may act in either a ductile

Figure 1.16. Adiabatic shear band formed in a compression specimen of Ta at a temperature of 77 K and a strain rate of $5.5 \times 10^3$ s$^{-1}$. Maximum stress occurred at a strain of $\sim 0.05$, and localization occurred at $\sim 0.23$. Material did not localize at room temperature with the same strain rate although maximum stress occurred at the same low strain. (Chen and Meyers, personal communication of unpublished data; collateral to work published by Meyers et al. 1995.)
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Figure 1.17. Fine scale deformation pattern shown in a Transmission electrons microscopy (TEM) metallograph of Al shocked to 9.0 GPa. (Reproduced from Grady et al. 1983, with permission from Kluwer Academic/Plenum Publishers.)

or brittle manner. Voids sometimes appear within the band, as shown in Figure 1.18, giving the appearance of having experienced tensile stresses while still at an elevated temperature. In contrast, short cracks sometimes also appear within a band (e.g., see Rogers 1983), suggesting that the critical stress had been experienced somewhat later when the material within the band had cooled.

Figure 1.18. Voids formed within a shear band in Ti-6Al-4V. (Reproduced from Grebe et al. 1985. Copyright 1985 by the Minerals, Metals and Materials Society and ASM International.)
1.1 Qualitative features of adiabatic shear bands

and achieved a hardened and more brittle state. In both cases, it is apparent that the locations of shear bands are also sites for possible future failures.

For applications in which a device is to be used repeatedly and must experience many service cycles, it is important that damage sites not be introduced either during its designed use or during its manufacture. For these cases it would be highly desirable to understand the conditions that produce adiabatic shear bands so that they can be avoided. This is generally not difficult for the service lifetime of a device in which plastic deformation itself is to be avoided. However, in modern high-rate manufacturing (forging, impact or electromagnetic forming, etc.), avoidance of the conditions that can produce adiabatic shear bands may be more problematic. Some knowledge of the mechanics of shear bands should allow the rational design of processes that avoid introduction of damage sites. In fact, guidelines for calculating material susceptibility to adiabatic shear have been reviewed (Semiatin et al. 1983 or Semiatin and Jonas 1984).

In contrast, in some industrial processes, such as drilling, cutting, shearing, or punching, as well as milling and machining, material failure by shear is an integral part of the process itself and occurs only once at a given location in the material. It would seem that optimum design of these processes could be more readily achieved with a thorough knowledge of shear mechanics. All the industrial processes mentioned so far have been developed largely by semiempirical means and of course are already in widespread use, but because of very high volumes worldwide, even small added efficiencies could translate into large cost savings.

Nevertheless, it is in ballistics and impact physics that the most immediate and dramatic payoff may be expected from the intelligent use of shear mechanics. In these cases not only is material flow and failure fundamental to the functioning of the service part, but if impact ceases without perforation, these processes must have been operating just beyond a limiting condition. Furthermore, as was suggested by the example of penetration, the extent and distribution of damage may be important in optimizing performance, as well as in minimizing the weight and expenditure of energy that are required for effective functioning. True optimization will not be achieved in the complex devices under consideration today without a full understanding of damage mechanics, of which adiabatic shear is a principal and often dominant example. Even design for crashworthiness of automobiles, aircraft, ships, and other vehicles (which generally requires consideration of lower levels of stresses and strain rates than those attained in ballistics) may well benefit from greater understanding of failure by adiabatic shear.
1. Introduction

1.1.4 Qualitative Mechanics

In a qualitative sense, the basic mechanics of adiabatic shear is easily understood. As plastic deformation develops, the isothermal flow stress generally increases with work hardening in most metals, and strain rate hardening may increase the flow stress still further. However, plastic work is converted mostly into heat, and as the temperature in the material increases, the flow stress usually tends to decrease. Thus, competing mechanisms are at work: work hardening and rate hardening tend to raise the flow stress, and thermal softening tends to lower the flow stress. Thermal softening almost invariably wins out over all other hardening mechanisms, so that if deformation continues long enough, eventually the material softens with increasing strain. The competing isothermal mechanisms and a typical adiabatic path are sketched in Figure 1.19.

Strain softening, if strong enough, becomes unstable so that small disturbances in the flow can accelerate, increasing the plastic work and heat generation locally, softening the material still further, and finally drastically altering the ability of the material to transmit shear forces. As the instability develops, a “postbuckling” structure forms, and the fully formed shear band takes shape. This process is generally referred to as localization. The reduced stress in the band now causes unloading in adjacent material where shearing virtually

![Figure 1.19](image)

Figure 1.19. Schematic stress–strain curves showing that the isothermal curves (solid lines) tend to lie at higher levels with increasing strain rate, and at lower levels with increasing temperature. The dashed line shows a typical curve for adiabatic loading. The curve starts along an isothermal path at a constant strain rate, but as plastic work and heating build up, the stress reaches a maximum and strain softening sets in.