SURVEYING INSTRUMENTS OF GREECE AND ROME

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CAMBRIDGE
UNIVERSITY PRESS

PUBLISHED BY THE PRESS SYNDICATE OF THE UNIVERSITY OF CAMBRIDGE
The Pitt Building, Trumpington Street, Cambridge, United Kingdom

CAMBRIDGE UNIVERSITY PRESS
The Edinburgh Building, Cambridge CB2 2RU, UK
40 West 20th Street, New York, NY 10011-4211, USA
10 Stamford Road, Oakleigh, VIC 3166, Australia
Ruiz de Alarcón 13, 28014 Madrid, Spain
Dock House, The Waterfront, Cape Town 8001, South Africa

http://www.cambridge.org

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First published 2001

Printed in the United Kingdom at the University Press, Cambridge

Typeface Monotype Bembo 10/13 *System* QuarkXPress™ [SE]

A catalogue record for this book is available from the British Library

ISBN 0 521 79297 5 hardback

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CHAPTER I

THE BASIC ELEMENTS

Surveying no doubt began at the humblest of levels, and for millennia evolved only slowly. Its functions would encompass the recording of the boundaries of plots of land, estimating their area and, if new-won land was being distributed, dividing it fairly; where irrigation or drainage was involved, ensuring that the gradient of water channels was adequate; in architecture, particularly of prestige buildings, establishing a reasonably horizontal level for foundations and sometimes, especially for religious monuments, the appropriate orientation. All these activities, as at every stage in the history of surveying, were based on geometry. At first this was doubtless entirely empirical and of the simplest kind; and at first the surveyors employed the simplest of tools. The real breakthrough to more complex requirements, to a deeper understanding of geometrical theory, and to procedures and instruments of considerably greater sophistication and precision, was due to the Greeks and Romans in the third and second centuries BC, and it is this revolution which forms the main subject of this book. But to understand its nature we need first to look at what it grew out of. A satisfactory investigation, unfortunately, is impossible simply because, before the treatises on the dioptra of Hellenistic Greece and the *Corpus Agrimensorum* of imperial Rome, our information is deplorably scanty.

For some topics, like measuring cords and plumb-line levels which hardly changed over centuries, the story is here continued to the end of the Roman period.

A. PRECURSORS OF THE GREEKS

The Greeks themselves always maintained that they learned the art of geometry – literally the measurement of land – from the Egyptians, who from time immemorial had recorded land boundaries so that, if obliterated by inundations of the Nile, they could be restored. The earliest record of this debt is found in the fifth century BC, when Herodotus wrote:[1]

[1] Herodotus, *Histories* II 109.

This king [Sesostris], they said, divided the country among all the Egyptians, giving each of them a square holding of the same size, and raised his revenue by levying an annual tax. Anyone who lost part of his holding to the river would come to the king and declare what had happened, and the king would send inspectors to measure how much land had been lost, so that henceforth the proper proportion of the assessed tax should be paid. This was the way, I think, in which geometry was invented and ultimately came to Greece; for the Greeks learned of the sundial and gnomon and the twelve divisions of the day from the Babylonians.

Five hundred years later the accepted story was little different:[2]

As the old report tells us, the first preoccupation of geometry was the measurement and distribution of land, whence the name geometry. The concept of measurement was discovered by the Egyptians; many plots of land disappeared in the floods of the Nile, and more when they receded, so that individuals could no longer identify their property. For this reason the Egyptians devised measurement of the land left by the Nile: they measure each side of the plot by cord or rod or other measures. Its usefulness to mankind being thus established, the practice spread everywhere.

Almost half a millennium later again, Cassiodorus repeats much the same tale.[3]

Such an origin for Greek geometry, at first in the literal and developing into the secondary sense, is inherently plausible. Significant contact with Egypt began when Greek mercenaries helped Psammetichus I recover his land from the Assyrians in about 660 BC, and increased with the creation of the Milesian trading post of Naucratis in the Delta in the late seventh century. At the same time Egyptian influence inspired Greece to adopt two artistic forms which were to have the profoundest consequences: architecture in stone and monumental sculpture.[4] The elements of practical geometry, later to generate equally revolutionary results, could very well have been transferred hand in hand with them.

Nobody could deny the proficiency of the Egyptians in some forms of surveying. It is well known how precisely the foundations of the Great Pyramid were laid out in terms of orientation, equality of sides and horizontal level – the latter achieved by cutting trenches in the

[2] This passage of Hero's survives in four very similar versions (*Geometry* 2, 23.1, *Geodesy* LXXII 9–18, CVII), the present translation drawing on all of them.

[3] Cassiodorus, *Variae* III 52.2.

[4] Boardman 1964, chapter 4.

rock along each face and filling them with water.[5] Long distances could be measured with considerable accuracy: two lines of boundary marks on either side of the Nile valley, intended to be of the same length, differ apparently by only 54 m over a length of 15 km.[6] Techniques of land surveying were doubtless entirely adequate for the somewhat limited purpose required of them. But, that said, Egyptian surveying instruments, from what little we know of them, were extremely simple. As we shall see, there were the ubiquitous plumb-line, level and square for building, the cord and rod for measuring, and possibly a crude precursor of the groma for laying out right angles. Although much has been made of the earliest known sighting instrument, the *merkhet*, it was applied not to surveying as such but to the ritual purposes of measuring the time or orientating a temple. It consisted of a split-palm leaf used as the 'backsight' and a plumb-line for the 'foresight'; if both were aligned on the celestial pole, they lay on a north–south line.[7] But there were obvious limits to the accuracy that such a hand-held device could attain.

Staggering achievements have quite unwarrantably been ascribed to Egyptian surveyors. Borchardt suggested that they levelled the course of the Nile for 1200 km from the sea to the First Cataract to establish a datum for the nilometers, the calibrated scales at the larger towns which recorded the height of the floods. He deduced that the gradient represented by the zero points on these scales averaged 1 in 14,440, as compared with the 1 in 13,700 of the river surface at low water as surveyed in modern times.[8] This theory is nonsense, for such a survey would be not only forbiddingly daunting but quite unnecessary. The zero point on each town's nilometer would be established from observations of low water recorded locally over many years; small wonder if the overall gradient which they preserve resembles the modern gradient.[9] Levelling was nevertheless practised. The Nile floods automatically watered low-lying fields beside the river, but to irrigate higher land further from the banks the floods were tapped by long diversionary canals running parallel to the river but at a shallower gradient.[10]

[5] Lehner 1983. [6] Montagu 1909, 80.

[7] Borchardt 1899; Lyons 1927, 135–6; Kiely 1947, 11–12.

[8] Borchardt 1906. 'Modern' of course means before the building of the Aswan High Dam. [9] For technical details see Bonneau 1986.

[10] For a useful survey of Egyptian irrigation see Oleson 2000.

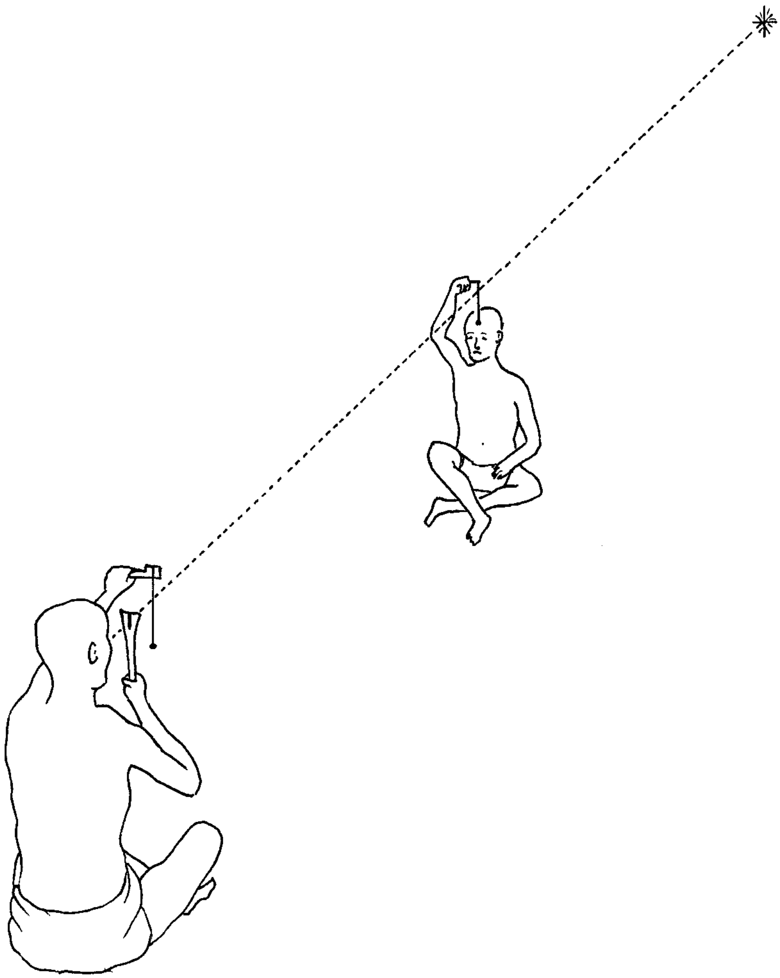

FIG. 1.1. The *merkhet* in use (after King 1955, Fig. 2).

Quite possibly the levelling was originally done with the water itself, by digging a ditch approximately along the contour and realigning or deepening it until the water flowed. If any instrument was involved, the only known candidate was the simple builder's level.

On the theoretical side, Egyptian astronomy was crude and empirical, and mathematics (and even arithmetic) was equally primitive. It was adequate to solve simple problems of quantity surveying such as estimating the number of bricks needed to build a ramp of given dimensions;[11] but it could teach the Greeks little apart from a few useful formulae for measurement. While Greece may indeed have been indebted to Egypt for the basic concept of land surveying and the most basic of equipment, the evolution of sophisticated instruments, which ironically began in Ptolemaic Alexandria, seems to have owed nothing to pharaonic Egypt.

The second potential source of inspiration for Greek surveying was Mesopotamia: Assyria to the north and especially Babylonia to the south. Here a very high level of mathematics, both in arithmetic and algebra and to some extent in geometry, had been practised for millennia; and the same is true of astronomy, although its predictive nature contrasted with the numerical and geometric approach finally achieved by the Greeks. In these spheres Greece undoubtedly learned far more from Babylonia than from Egypt. But the pupillage was gradual, doubtless because, before Alexander's conquests, there was little direct contact between the two cultures. Indeed Babylonian mathematical astronomy reached its highest level under Greek and later Parthian rule, between 311 BC and AD 75. Herodotus remarks, in the passage quoted above, that Greece learned of the sundial, gnomon and twelve-hour division of the day from Babylon, which is likely enough; and the claim that the philosopher Anaximander (*c.* 610–545 BC) discovered the gnomon should no doubt be taken to mean that he merely introduced it to Greece.[12] Some Babylonian astronomical practices, we shall see (Chapter 2.B), were adopted by the Greeks in the course of the fourth and third centuries, but it was only in the second, when Hipparchus evidently consulted Babylonian records at first hand, that the link became close and the golden age of Greek mathematical astronomy began.[13]

The theory of surveying and its cognate sciences, then, at least as they evolved into more advanced forms, owed more to Babylon than to Egypt. As for the practice, little is known of Mesopotamian methods. In Assyria about 690 BC Sennacherib built a major aqueduct at least 50 km long for supplying water to Nineveh, which

[11] Neugebauer 1969, chapter IV.

[12] Kirk, Raven and Schofield 1983, 100–1, 103–4.

[13] For Babylonian mathematics and astronomy and their transfer to the Greeks see Neugebauer 1969, chapter II; Dicks 1970, 163–75; Jones 1991.

wound along the hillside and incorporated bridges just like its later Roman counterparts.[14] In Babylonia, with its need for perennial irrigation, the network of canals was even more complex than in Egypt.[15] For levelling, the medieval surveyors of Islamic Iraq used the *mizan* which, given that the region coincided with the former Babylonia and the requirements were identical, they may very well have inherited, like their units of length, from earlier civilisations. In essence – more detailed discussion must await Chapter 12 – the *mizan* consisted of a cord held between two graduated vertical staves. The ends of the cord were raised or lowered until a plumb-line device at the centre showed that they were level. Their relative positions on the graduated scales gave the difference in height between the bases of the two staves. Measurement was by cord and rod; and irregular plots of land on Babylonian survey maps, subdivided into right-angled triangles, rectangles and trapezoids whose dimensions and areas are indicated,[16] show an approach to land surveying similar to that of the Greeks.

The final oriental source of inspiration for the Greeks was very likely Persia, where there was a long-established tradition of tapping underground aquifers and conducting the water to the surface by means of qanats or tunnels. This technology spread to Egypt, and especially the Kharga oasis, at an early date. The first significant Greek aqueduct tunnel was built about 530 BC by Polycrates, the tyrant of Samos. It was closely followed by another at Athens, and at about the same time similar counterparts appeared in Etruria. Persia first impinged directly on the Aegean world with Cyrus' conquest of Asia Minor in the 540s BC. At this stage Polycrates was in close alliance with Egypt, but in 530 abruptly changed his allegiance to Persia. This period therefore seems much the most likely occasion for the transfer of the specialist technology of surveying and driving tunnels, whether before 530 indirectly via Egypt or afterwards from Persia. At least in later times the qanat was levelled by means of a suspended sighting tube, and it is possible, though very far from proved, that this gave rise in later generations to the standard Greek dioptra and the standard Roman libra. This difficult question is debated more fully in subsequent chapters.

[14] Jacobsen and Lloyd 1935. [15] Oleson 2000.

[16] Kiely 1947, 12–13. On Babylonian maps and plans see Millard 1987.

B. MEASURING DISTANCES

Accurate measurements of length, and especially of long lengths, were surprisingly difficult to achieve, and figures worked out geometrically would inevitably reflect any inaccuracies in the measured distances on which they were based.[17] In Greek surveying the normal measuring device was the cord, made of a variety of fibres. The sort most commonly found was the *schoinion* which, according to the derivation of the name, was strictly of twisted rushes but, one suspects, was more often made of other substances.[18] Not only did the word denote a measuring line in general, but also the specific distance of 100 cubits,[19] which presumably reflects a standard length of cord. This was subdivided into 8 *hammata* or knots of 12½ cubits apiece, no doubt because it was knotted at those intervals.[20] The *schoinion* was well entrenched in Ptolemaic Egypt. The *aroura*, the standard unit of area for land, was one *schoinion* square, and the term *schoinourgos* was sometimes applied to the land surveyor.[21] Another fibre employed for cords was esparto, whence *spartos* (which is found in Hero, but only for cords whose precise length was not of importance, such as plumb-lines and for laying out straight lines on the ground) and *sparton* and *spartion* (words which are applied to measuring cords by other sources). The flax measuring cord (*linee*) is encountered in the second century BC in Boeotia[22] and in the Talmud,[23] and **al-Karaji 1** specifies a 100-cubit cord of well-twisted flax or silk, the latter no doubt an Islamic alternative. We will meet hair and hemp cords in a moment.

The problem with any fibre cords is that, unless very well pre-tensioned and protected from damp, they are liable to shrink or stretch according to their moisture content. Official cord-keepers and

17 For an overview of Greek measuring devices see Coulton 1975, 90–1.

18 For references over and above those given in the notes to this section, see LSJ under the words in question.

19 To be distinguished from the very much longer *schoinos* which varied between 30, 40, 48 and 60 stades.

20 Shelton 1981, citing a number of papyri. Knots are visible on the surveying cord depicted in a well-known pharaonic fresco (reproduced by Lyons 1927, f.p. 132; Kiely 1947, Fig. 1; Dilke 1971, 49). The *amma* of 40 cubits mentioned by Hero, *Geometry* 4.12, 23.14 has not been found in papyri.

21 As was the term *harpedonaptes*, 'cord-fastener'.

22 *IG* VII 3073.128.

23 Talmud, *'Erubin* 58a, and also rope made of palm fibre.

cord-stretchers are attested far back in pharaonic Egypt.[24] Hero was well aware of this failing, and several times insists on 'a cord (*schoinion*) that has been well tensioned and tested so that it will not stretch or shrink' (***Dioptra* 20**).[25] Elsewhere he describes how to prepare cords for use in automata by a process that sounds equally applicable to measuring lines:[26]

The cords must not be capable of stretching or shrinking, but must remain the same length as they were to start with. This is done by passing them round pegs, tensioning them tightly, leaving them for some time, and tensioning them again. Repeat this a number of times and smear them with a mixture of wax and resin. It is then best to hang a weight on them and leave them for a longer time. A cord thus stretched will not stretch any more, or only a very little.

A Byzantine treatise on land surveying of uncertain date may also preserve features from an earlier period:[27]

The cord which you intend to make into a 10- or 12-fathom measure should not be of hair, because this has an unreliable quality and always gives a misleading measurement. If it is partially, or above all totally, soaked in dew it immediately shrinks and shortens by a fathom; then, on drying out again and stretching, the 10 fathoms, from the slackening and extending, becomes 11, and the cord's accuracy remains misleading. Instead, the cord for measuring should be of hemp, thick and firm. First make short pegs, one spade-shaped with a flat iron blade underneath to cut and mark the earth around each cord, the other a sharp iron for fixing and positioning in the mark left by the first. Both of these marker pegs have solid iron rings into which the ends of the cord are tied and sealed with a lead seal [to prevent fraud by shortening the cord]. At each fathom along the cord a thick tuft is hung to indicate the fathoms . . . If the pegs tied to the measuring cord are [too] long, they can be tilted by pulling on the cord, and each cord length can gain 5 spans or half a [fathom] or even more.

There were two alternatives to unreliable fibre cords. One was the measuring chain (*halysis*), which Hero twice mentions as a substitute (***Dioptra* 34** and (not translated) 23). Again, Rabbi Joshua b. Hananiah, a contemporary of Hero, said, 'You have nothing more suitable for

[24] Dilke 1971, 21.
[25] Similarly in *Dioptra* 23 and 25, not translated in Part III.
[26] *Automata* II 4–5.
[27] Schilbach 1970a, 51.12–52.2; see also Dölger 1927, 83–4 and Schilbach 1970b, 28–9.

measuring than iron chains, but what can we do in the face of what the Torah said?',[28] referring to the Jewish law which specified that Sabbath limits must be measured only with ropes exactly 50 cubits long. From the paucity of these references, the chain was evidently much rarer than the cord, no doubt because of its cost and perhaps because of its weight.

The other alternative was the measuring rod (*kalamos*). Originally made of a reed, as its name implies, it could also be of wood and in Ptolemaic and Roman Egypt might be either 5 or 6⅔ cubits long.[29] The latter, also known as the *akaina*,[30] corresponds in length to the wooden ten-foot rod (*decempeda* or *pertica*[31]) of the Roman surveyor,[32] which was furnished at the end with bronze ferrules marked in digits or inches for small measurements and flanged to butt neatly against its neighbour.[33] Since wood expands and contracts very little along the grain, rods would give a much more accurate result than cords, and were evidently standard equipment for the architect and builder. But for the surveyor and the longer distances over which he operated, rods would be vastly more tedious to use. None the less, there is hardly any evidence that measuring cords were employed at all by Roman surveyors,[34] who seem rather to have relied on ten-foot rods used in pairs, one leapfrogging the other. The same Byzantine treatise also speaks of fathom rods of wood or reed with a lead seal at either end to deter malpractice.[35]

It is entirely feasible that Greece should have learned the techniques of land measurement from the Egyptians and, by way of their colonies in Italy, passed them on to the Romans. But, once again, the potential contribution of the Babylonians should not be ignored. A relief from Ur dating to about 2100 BC depicts a god commanding the king to build a ziggurat and holding what appear to be a coiled measuring cord

[28] Talmud, *'Erubin* 58a. [29] Hero, *Geometry* 4.11, 23.13. [30] Hero, *Geometry* 4.12.

[31] Balbus, *Explanation* 95.6–7. The *pertica* might on occasion be 12, 15 or 17 feet according to local circumstances: Hultsch 1866, 136.6.

[32] Who was consequently sometimes known as the *decempedator*: Cicero, *Philippics* XIII 37.

[33] Examples found at Enns in Austria are illustrated by Lyons 1927, facing p. 140 and Dilke 1971, 67, others from Pompeii by Della Corte 1922, 85–6.

[34] The tombstone of an *agrimensor* from Pompeii does depict a cord alongside two rods, but it could be for setting out a straight line rather than for measuring (Adam 1994, 10). [35] Schilbach 1970a, 51.29–32.

and a measuring rod,[36] reminiscent of the angel with a flax cord and rod whom Ezekiel saw in a vision in Babylonia in the sixth century.[37] The surveyor is referred to in land charters as 'the dragger of the rope'.[38]

For really long distances – too long to be measured by cord – there was the option of counting paces. On the staff of Alexander the Great during his campaigns were expert *bematistai* whose job it was to count their paces as they marched and to note the direction of travel and the names of places passed, so that from their records outline maps could be compiled and descriptions of the routes published. We know the names of a few: Diognetus, Baeton and Amyntas.[39] Their results were necessarily approximate but, as Sherk remarks, we should not underestimate their abilities.[40] It was probably their successors in the service of the Ptolemies who measured the overland distances required by Eratosthenes in his estimate of the circumference of the earth (Chapter 7).

Surveyors recorded their field measurements on wax tablets or papyrus (***Dioptra*** 7), and for arithmetical calculations they no doubt used the abacus, which was well known throughout the ancient world.[41]

C. ORIENTATION AND RIGHT ANGLES

Temples and town grids and even land boundaries sometimes needed setting out to a particular orientation. It was not difficult to establish a north–south line. An approximation could be found by simply observing the stars, but not so easily as today because Polaris was then remote from the celestial pole (about 18° in 1000 BC, about 12° in AD 1). The nearest bright star to the pole was Kochab, β Ursae Minoris, about 6° distant in 1000 BC but moving further away. It was better to use the sun, observing the shadow of a vertical gnomon and marking the point where it appeared to be at its longest. Better still, a circle was traced around the base of the gnomon, the points were marked where the tip of the shadow touched the circle before and after noon, and the result-

36 Woolley 1925, 398 and pl. xlviii.

37 Septuagint, Ezekiel 40.3.

38 Lyons 1927, 137.

39 Pliny, *Natural History* VI 61; Athenaeus, *Learned Banquet* 442B.

40 Sherk 1974, 535n.

41 Dilke 1987a, 21–2.

ing angle was bisected.[42] Similarly, the directions from a stake of the rising and setting of a star could be marked by planting two poles and again bisecting the angle. Since the horizon is rarely entirely level, a more accurate result could be obtained by creating an artificial horizon in the form of a temporary circular wall whose top was levelled by water and, sighting from a pole at the centre, marking the points on the wall where a star rose or set.[43] This was very likely how the pyramids were orientated; as is well known, the sides of the Great Pyramid diverge from true east–west and north–south by a maximum of 5′ 30″ and a minimum of 1′ 57″.[44]

To set out a right angle without an instrument, various methods were possible with cords. The properties of the triangle whose sides are multiples of 3, 4 and 5 were well known (if not proved) long before Pythagoras. Euclid showed, as had probably been accepted earlier, that a triangle contained by a semicircle is right-angled, and that lines drawn between the centres of two overlapping circles and between the intersections of their arcs cross at right angles. These facts are mentioned by Balbus, though it is not clear if they were applied in the field.[45] A relatively small set square could be laid on the ground, its sides extended by cords and pegs, and the diagonals of a resulting rectangle measured to check that they were the same.

D. MEASURING HEIGHTS

Civilian surveyors, unless motivated by pure curiosity, rarely needed to discover the height of existing structures or objects; but in warfare it was often necessary for a besieging force to try to scale a city wall by ladder or by siege tower. To construct them to the right size, the height of the wall had to be discovered and, since it would normally be suicidal to attempt to measure it directly, more devious methods were developed. The simplest was to count the courses of brick or stone, estimate or surreptitiously measure a typical course, and multiply it out. We first hear of this ruse in 428 BC when the besieged Plataeans, wishing to break out, counted the bricks in the Peloponnesians' siege

[42] This method is described by Vitruvius I 6.6; Proclus, *Outline* III 23–4; and Hyginus Gromaticus, *Establishment* 152.4–22, who adds another complex method, more academic than practical, based on solid geometry.

[43] Edwards 1985, 246–7.

[44] Edwards 1985, 99.

[45] Balbus, *Explanation* 107.12–108.8.

wall.[46] In 212 BC Marcellus' long and hitherto fruitless siege of Syracuse was brought to an end when a Roman soldier escorting a conference party outside the wall of Epipolae noticed how low the defences were and furtively counted the courses; his information allowed Marcellus to construct ladders and attack this weak point when the Syracusans were distracted.[47] Later generations would normally use dioptras to obtain the height (Chapter 3.H), but at this date they were probably not available, and in any event the wall at this point was close to the sea, allowing no room to use them.[48] As late as AD 537 the Goths besieging Rome – who could hardly be expected to have dioptras at their command – counted the courses of the wall preparatory to building siege towers.[49]

Vegetius, the military source of the late fourth century AD, offers two alternative methods of finding wall heights. One, which he fails to explain properly, involves tying a fine thread to an arrow which is shot at the top of the wall, and somehow deducing the height from the length of thread. The other is to clandestinely measure the shadow of the wall and at the same time set a ten-foot rod vertically in the ground and measure its shadow.[50] The relationship between the rod and its shadow will be the same as that between the wall and its shadow. Polybius, writing about 150 BC on the same subject, says that 'the height of any object standing vertically on a level base can be taken from a distance by a method that is practicable and easy for anyone who cares to study mathematics'.[51] One might have expected both authors to mention the dioptra, for both wrote within its lifespan. In the case of Polybius his vague language might refer either to the shadow stick or to the dioptra; in the case of Vegetius there is, strangely enough, no good evidence that the Roman army ever used the dioptra.

The practice of measuring heights by the shadows they cast is of considerable antiquity. Its discovery (at least by the Greeks) is traditionally ascribed to Thales, the philosopher of Miletus (624–546 BC?), who is said to have been the first Greek geometer and to have visited Egypt. Miletus' close links with Naucratis make such a journey plausible enough. According to one later legend Thales measured the heights of the pyramids by their shadows at the time when his own shadow

[46] Thucydides, *History* III 20.3–4.
[47] Polybius, *Histories* VIII 37.2.
[48] For the site, Parke 1944.
[49] Procopius, *Gothic Wars* I 21.3–4.
[50] Vegetius, *Epitome* IV 30.
[51] Polybius, *Histories* IX 19.8.

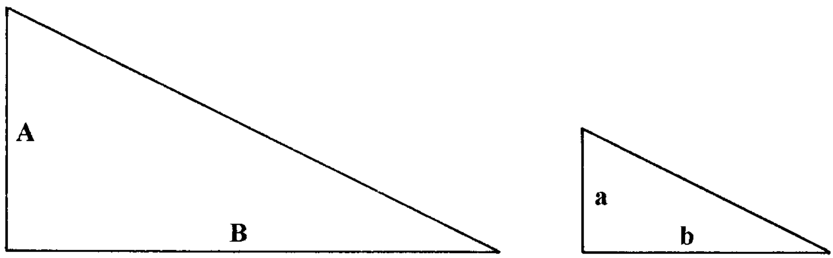

FIG. 1.2. Similar triangles.

equalled his own height, that is when the sun stood 45 degrees high. Another version makes him understand the wider truth that, wherever the sun may be, the heights of all vertical objects are in the same proportion to the lengths of their shadows at the same time of day.[52] The concept remained well established in the geometer's basic repertoire. Hero explains it simply and clearly: to find the height of a tall column, set up a 3-foot rod near by, and measure the shadows of the rod and the column. If the rod's shadow is, say, 6 feet long and the column's is 100, the column is 50 feet high.[53]

The principle involved, utterly simple but lying at the root of ancient surveying, is that of similar triangles. If the angles of two triangles are the same (in this case because the objects are vertical and the sun is at the same height), then the lengths of their sides are in proportion. Thus **a** is to **b** as **A** is to **B**, and since **a**, **b** and **B** are known, **A** is easily worked out.

Normally heights were found from distances, but it was equally possible to find distances from heights. This discovery, without shadows, was also attributed to Thales who 'demonstrated the distance of ships out at sea'.[54] Later Greek writers did not know exactly how he did this, but assumed that he used similar triangles. There are a number of possible methods which differ only in detail. If, for example, he stood on a tower of known height **A** above the sea, and positioned a vertical rod **a** so that the sight line from its apex to the ship's hull touched the edge of the tower, then **a** and **b** could be measured and the required distance **B** was found by multiplying **A** by **b** and dividing by **a**.

[52] Kirk, Raven and Schofield 1983, 76–86.
[53] Hero, *Stereometry* 2.27.
[54] Kirk, Raven and Schofield 1983, 85.

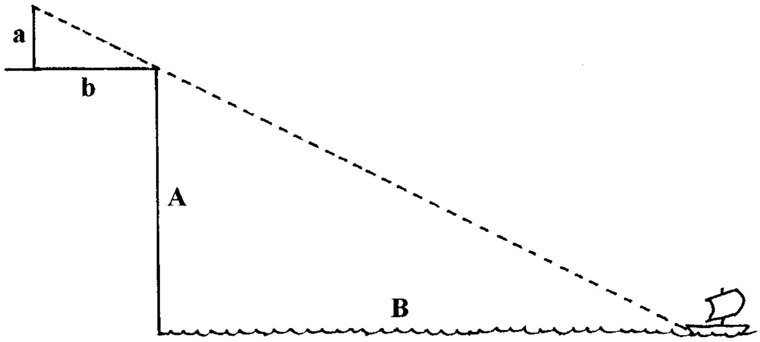

FIG. 1.3. Thales' demonstration of the distance of a ship.

This kind of straightforward geometry underlies most of the exercises later carried out with the dioptra. It also recurs in a pair of simple devices for discovering heights, recorded by late Latin sources, in the form of light triangular wooden frameworks where the base is held horizontal and the object is sighted along the hypotenuse. A snippet of Vitruvius Rufus preserved in the *Corpus Agrimensorum* (**Source 1**) deals with a right-angled isosceles triangle. Nothing is known of the author; but while it is very unlikely that he is *the* Vitruvius, it is curious that the former's phrase 'lie flat on your teeth' (*decumbe in dentes*) is almost exactly matched by the latter's instruction *procumbatur in dentes* when looking for mist rising from the ground as an indication of the presence of water.[55] A fragment in the *Mappae Clavicula* (**Source 2**), which might also derive from the *Corpus*, describes a more sophisticated right-angled 3–4–5 triangle.

Finally the shadow stick, or gnomon as it is more properly called in this context, was also used for measuring the altitude of the sun by the same process of comparing the height of the rod with the length of its shadow. This became a standard method of determining latitude, taking the reading at noon on the summer solstice. At first the result was given cumbrously as the ratio between gnomon and shadow; for example Pytheas in the fourth century BC recorded the latitude of Marseille as 120:41⅘. When from the second century BC the system of

[55] Vitruvius VIII 1.1; somewhat similar phrases are also found in Pliny, *Natural History* XXXI 44, Palladius, *On Agriculture* IX 8, *Geoponica* II 5.11 and Cassiodorus, *Variae* III 53.

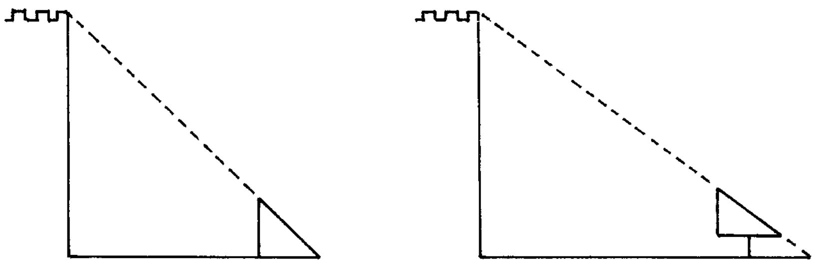

FIG. 1.4. Height triangles of Vitruvius Rufus and the *Mappae Clavicula*.

360 degrees was adopted (Chapter 2.B), this ratio could be converted into the angle of the sun below the zenith, and adding the obliquity of the ecliptic gave the latitude in degrees. Pytheas' ratio for Marseille works out at 43° 11′, compared with the modern value of 43° 17′.[56]

E. LEVELLING

Of the two fundamental methods of finding a horizontal, the water level was little favoured in the ancient world. It is true that, as we remarked, the Great Pyramid was levelled by water, and there are other instances where the Egyptians probably flooded a complete building site to establish an overall level; but this approach was hardly practicable in Greece.[57] Instead, for setting a vertical and hence a horizontal at right angles to it, the principle of the plumb-line ruled supreme. The tools which became traditional to the carpenter and builder in many cultures first appear in Egypt: the square, the rigid rule, the string, the A-frame level for horizontals and its counterpart for verticals. Of these it is the A-frame level which concerns us. A right-angled isosceles triangle with a cross-bar is made of bronze or wood (sometimes strapped with bronze at the joints), and a plumb-line is hung from the apex. When the line coincides with a vertical mark scribed on the centre of the cross-bar, the feet are at the same level.

The Greeks borrowed the device at an early date and used it extensively.[58] At first it was called the *staphyle* or 'grape', referring to the

56 Strabo I 4.4, II 1.12, 5.8, 5.41. For discussion, see Dicks 1960, 178–9. This conversion into degrees uses the correct value of the obliquity for that period, not the approximation (usually 24°) then current.

57 Coulton 1982, 46.

58 Martin 1965, 188–9.

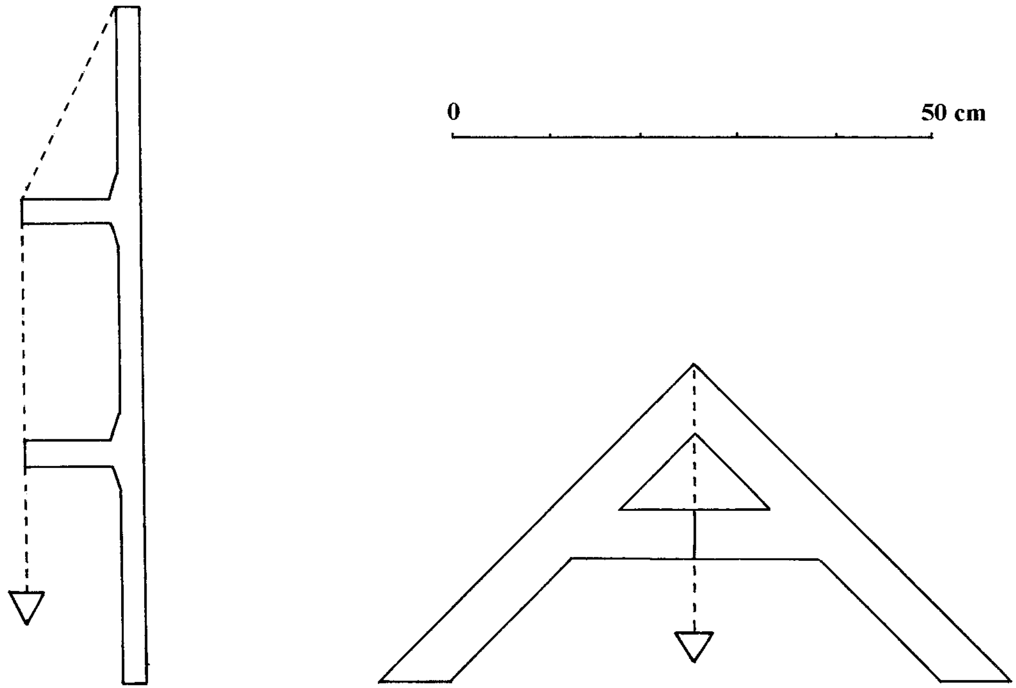

FIG. 1.5. Egyptian A-frame level for horizontals and plumb-line for verticals, *c.* 1300 BC (based on Glanville 1942, Pl. 22).

plumb bob, but it then acquired its usual name of *diabetes*, 'strider', after its two equal legs: for the same reason the term was also applied to a pair of compasses. Homer makes Admetus' horses identical in height 'according to the *staphyle*'.[59] A later scholiast explains this as 'the mason's *diabetes* which measures the horizontal and the vertical at the same time', a definition displeasing to another commentator who adds 'but the *diabetes* only measures the horizontal'.[60] *Pros diabeten*, literally 'by the level', became the standard term for 'horizontal' and crops up frequently in Hero's *Dioptra*. Theon of Alexandria (**Source 4**) mentions the same device under the alternative name *alpharion*, alpha-shape; and *alphadion* and *alpha* are also found in late sources.[61] The A-frame level was likewise extensively used by the Romans,[62] who called it the *libella* (for the derivation of which see Chapter 4, and for references **Source 44**); and *ad libellam* was the precise equivalent in Latin of *pros diabeten*, level or horizontal.

The A-frame level was clearly the standard tool for levelling the foundations and wall courses of buildings under construction. For most purposes it was no doubt entirely adequate, but for large structures its accuracy necessarily depended on four factors. If it was not very precisely made it could mislead. Even given precision of manufacture, the smaller it was, the less accurate the results. Its plumb was liable to sway in the wind. And even in a total calm it would be impossible to align the plumb-line with absolute precision over the mark on the cross-bar, given that both had an appreciable thickness. None the less, as the inscriptions attest, it was used on prestige buildings, where its deficiencies are only revealed by detailed measurement. The deliberate curvature of the stylobate of the Parthenon, for instance, is well known: the whole floor is convex, rising about 10 cm on the sides (69.51 m long) and 7 cm on the ends (30.86 m). The workmanship which achieved this refinement is properly admired. What is less well

59 Homer, *Iliad* II 765.

60 *Scholia in Iliaden* I 130. The *diabetes* is found not uncommonly in literature and in epigraphic specifications for public building works, e.g. *IG* II^2 1668.10 for the Piraeus arsenal in 347/6 BC.

61 From *alpharion* comes the modern Cretan *alphari*. For *alphadion* (whence modern Greek *alphadi*) and *alpha* see Eustratius, *Commentary on Nicomachean Ethics* 322.18 and 74.2.

62 Adam 1994, 41–2, Figs. 48, 51–2, 79, 81–3 including variations on the basic theme.

known is the fact that the platform on which the stylobate rests, clearly meant to be horizontal, is not: it rises to a peak at the south-west corner where it is about 5 cm too high.[63] The fault presumably lay in the levelling devices employed.

The Parthenon was built in the fifth century BC when Athens was at the pinnacle of her greatness. Not only is it considered the acme of Greek architecture, but it was intended as a deliberate manifestation of her glory, and one may assume that its architects drew on all the best and latest technology. Yet its foundation slopes at an average of 1 in 1400. In architectural terms this failing, being quite undetectable to the observer, is hardly a serious one. But if the same instruments were used for levelling an engineering project, similar inaccuracies might not be so immaterial. The ultimate test would come with the extremely shallow gradients of some Roman aqueducts, for which inaccuracies of this order would be quite unacceptable. With Hellenistic aqueducts, as we shall find, gradients were very much steeper and such errors might be tolerable, as they would with most irrigation channels.

Any level like the *diabetes*, therefore, which depended on a simple plumb-line was fine for the builder and passable for the Greek engineering surveyor, provided he did not require shallow gradients; it was not adequate for the Roman aqueduct surveyor whose work demanded a precision instrument. In fact there is no certain evidence whatever – even in the *chorobates*, which we will shortly meet – that the ancient world used the simple plumb-line level for any purpose other than building construction. This is no doubt our ignorance, for it seems highly likely that, in the absence of anything better, such levels were indeed used for irrigation work at least.[64] It is not difficult to visualise the surveyor placing one on a plank, which he adjusts until the level shows it to be horizontal. He squints along the top of the plank at a pole held by an assistant, whom he instructs to mark the pole where his line of sight meets it. The difference in height between the mark and the plank represents the rise or fall of the ground. It sounds easy; but anyone who tries to sight a relatively distant object along a straight

[63] Carpenter 1970, 116.

[64] For the irrigation systems of the Negev and the Maghreb, more localised than in Egypt and Mesopotamia and originated by indigenous peoples even if further developed under Roman rule, see Evenari et al. 1982, Lindner 1987, Birebent 1964 and Shaw 1984.

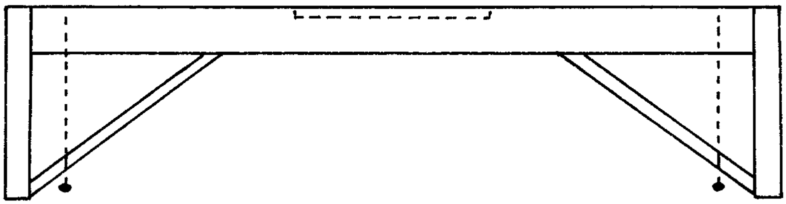

FIG. 1.6. The chorobates.

edge will discover how imprecise the operation is, although it is much improved by the addition of simple sights such as ring-headed nails driven into the plank. This scenario is purely speculative; but it does have something in common with our oldest description of instrumental levelling by Philo of Byzantium (**Source 10**).

If such a method might suffice for simple aqueducts, irrigation channels and drains, it would not suffice for more advanced aqueducts. These could only be surveyed with advanced instruments which likewise worked on the plumb-line principle, but differed from the A-frame level in two important respects. First, they had proper sights; and second (certainly the dioptra and probably the libra) they were suspended to act as their own plumb bobs, and could therefore be reversed to check the sightings taken. These matters will be explained in later chapters. For the moment, suffice it to mention the possibility that about 530 BC both the Greeks and the Etruscans inherited from Persia the suspended sighting tube for levelling tunnels, and that this was responsible for the survey, astonishingly precise for so early a date, of the famous tunnel on Samos and a few of its contemporaries.

This brings us on to Vitruvius and his famous – one is tempted to say infamous – chapter on the chorobates (**Source 3**). Whatever detailed reconstruction one prefers, notably in the cross-pieces bracing the legs, the broad outline is perfectly clear. Because Vitruvius describes the chorobates in detail, it was evidently a novelty to his Roman readers. Yet, because he claims that it was more accurate than the dioptra and the libra, it is almost universally assumed to have been the principal instrument for levelling Roman aqueducts. It must be said at once, however, that as a serious instrument for surveying shallow gradients it has very few qualifications indeed.

Firstly, at 20 feet long and on legs, it resembled a lengthy but narrow trestle table. To avoid sagging in the middle its straight-edge must have been of considerable depth and therefore weight. It can only have been levelled by adjusting wedges under its feet. It would be exceedingly, one might even say impossibly, cumbrous to use in the field, and especially in rough terrain.[65]

Second, the very length of the straight-edge ought to be an aid to precision: the further apart the sights the better. Yet Vitruvius does not even hint that it possessed sights at all, or that readings were taken on a calibrated surveyor's staff, both of which are essential prerequisites for accurate levelling and both of which were already known to Philo (**Source 10**) two centuries earlier. In view of this deficiency, Montauzan suggested that two or more instruments were set up touching each other end to end and made to leapfrog each other in continuous horizontal steps, the difference in height being recorded each time.[66] The theory may be appreciated, but the practice, on aqueducts of any length, is totally beyond belief.

Third, the chorobates is prey to the defects inherent in every plumb-line level: the impossibility of exactly aligning the string on the mark, and movement of the bobs in the wind. Vitruvius, recognising this latter problem, supplies a water level as well. But a wind that will swing the plumb bobs will also ruffle the surface of the water; and why is the trough only five feet long when it could be four times the length? Let us take the chorobates as 6 m long and 1.5 m high, and the trough as 1.5 m long. If the strings are in contact with the frame, friction will prevent them from hanging exactly vertical; if they are clear of the frame they are more liable to swing and more difficult to align to the marks. If at the lower end their centre line is half a millimetre to one side of the centre line of the mark, the top of the straight-edge (always assuming total precision of manufacture) will slope at 1 in 3000. If the water in the trough is half a millimetre higher at one end than the other the slope will be the same. If the error in either case is one millimetre, the slope is 1 in 1500. This point will be picked up again in Chapter 9.C in connection with aqueduct surveying. Adam constructed a quarter-size replica (1.5 m long) to which he added sights, and tested it in the field (using a staff) against a modern instrument. Over a traverse of 51.3

[65] Smith 1990–91, 60.

[66] Montauzan 1909, 166, followed by Callebat 1973, 140–1.

m the chorobates was in error by 4 cm, equal to a slope of 1 in 1282.5.[67] Even allowing for the relatively small scale, this is not an impressive result, especially with sights and staff.

In short, the chorobates was in essence a glorified builder's level. As Vitruvius himself says, it was fine for levelling a single position such as, we may imagine, a temple platform. But for projecting a level over a long distance, as aqueduct surveying requires, it was not good. As Ashby rightly remarked, 'Vitruvius may be guilty of an architect's prejudice in favour of an instrument more useful to a builder than to a field surveyor.'[68] But, even if Vitruvius was misled or simply wrong, the question of his source remains to be answered. Although it is always assumed that no further information on the chorobates is available, in fact something of its history can be recovered.

A Greek origin is proved by the exceedingly rare name, which translates as 'land-ranger' or 'land-pacer', a strange soubriquet for a levelling device. The associated verb, *chorobatein* (as rare as the noun), means to measure land by pacing.[69] It is found in this sense in the Old Testament as translated into Greek at Alexandria about 200 BC, where Joshua sends men to measure the Promised Land before it is divided among the Children of Israel.[70] It is used in a papyrus of 248 BC of workers who might be measuring or might merely be inspecting a vineyard.[71] It is found once in Hero (**Dioptra 12**) apparently in the sense of 'taking levels with a dioptra'.[72] From this flimsy evidence one might deduce that the meaning of the noun, originally 'land-pacer', was narrowed to 'leveller'.

This is undoubtedly the sense – builder's level – in which it was used by Theon of Alexandria (**Source 4**), who says that the *diabetes* or *alpharion* for levelling a foundation resembles Carpus' chorobates. Who was this Carpus, and when? Only one man of that name is known who had scientific and technical interests, and we may be sure that he is the right one. He was a mechanic from Antioch in Syria who wrote on

[67] Adam 1982, 1029; cf. Adam 1994, 18–19.
[68] Ashby 1935, 37.
[69] Hesychius, s.v. *chorobatein*.
[70] Septuagint, Joshua 18.8.
[71] *P. Cair. Zen.* 59329.
[72] The noun may also occur on a late (Christian) tombstone from Corycus in Asia Minor, describing the profession of one Sergius (*MAMA* III 694). He is usually taken to be a land-surveyor, but the word has a short *o*, not a long one, and, if it is correctly spelt, means that he was a chorus-dancer.

astronomy and geometry;[73] not a great deal is recorded about him, and even his date is not certain. Proclus reports that in his *Astronomical Treatise* Carpus discussed (apropos Euclid) whether problems come before theorems, a matter on which Geminus held contrary views.[74] It is sometimes assumed that Carpus was criticising Geminus, who lived (it is usually accepted: see Chapter 7) in the first century BC, and that therefore Carpus was contemporary or later.[75] If so, he might be too late for Vitruvius. But all Proclus is saying is that the two men held different opinions, and Carpus could just as easily be earlier than Geminus.[76]

There are indeed two suggestions that this was so. Pappus notes that, according to Carpus, Archimedes wrote only one book on practical mechanics (on the construction of a planetarium) because he refused to allow external applications to sully the purity of geometry and arithmetic. But, Pappus goes on, 'Carpus and some others did make use of geometry for certain practical techniques, and with good reason; for geometry, which by its nature can foster many techniques, is in no way injured by its association with them.'[77] It sounds as if Carpus was a pioneer in applying geometry to instruments. If so, it must have been not long after Archimedes' death in 212 BC and before about 150, by which time, as we will see, instruments for astronomy and for terrestrial surveying, all governed by geometry, were well established. The other indication is a short list of mathematicians who constructed curves for squaring the circle:[78] Archimedes (*c.* 287–212), Nicomedes (a little older than Apollonius[79]), Apollonius of Perge (probably *c.* 260–190/180[80]), and Carpus. Since the first three names are in chronological order, Carpus should have been younger than Apollonius; but not much younger, because squaring the circle was a preoccupation of earlier Hellenistic mathematicians, not of later ones.[81] On two counts, therefore, it is likely that Carpus was active in the first half of the second century BC.

[73] Pappus, *Collection* 1026; Proclus, *Euclid* 125.25, 241.19, *Republic* 218.22.

[74] Proclus, *Euclid* 241.19– 243.11.

[75] So Neugebauer 1975, 943; Aujac 1975, LXIII; Heiberg 1919.

[76] As Tannery 1887, 147n saw.

[77] Pappus, *Collection* 1026.

[78] Iamblichus quoted in Simplicius, *Physics* 60.15 and *Categories* 192.19.

[79] Fraser 1972, II 610.

[80] Fraser 1972, I 416.

[81] Heath 1921, I 220–32 knows of no mathematician after Apollonius/Carpus who contributed to the problem.

As I have tried to show elsewhere,[82] Vitruvius knew little at first hand about aqueducts or surveying, and his chapters on aqueducts are very largely derived from Hellenistic Greek sources which he by no means fully understood. His chapter on the chorobates is entirely typical; and it smacks, moreover, of the language an inventor might use in publicising his own work. It seems possible that Carpus in one of his writings had sung the praises of his chorobates as a builder's level and, at least potentially, as a level for the aqueduct surveyor; and that Vitruvius, who had never set eyes on the thing, merely translated or summarised his words. Carpus' evident interest in and knowledge of Archimedes' work would account for the reference to Archimedes' theory about water surfaces, and an advertising motive would explain the derogatory reference to the dioptras and libra, which we can well believe were less appropriate for levelling masonry but were assuredly superior in levelling aqueducts.[83]

Because Theon (or his source) clearly expected his readers to know what Carpus' chorobates was, we may assume that it had to some extent caught on as a builder's level. As a surveyor's level, we have no idea whether it remained merely a gleam in Carpus' eye or did find use on Greek aqueducts with their relatively steep gradients; it would no doubt be preferable to the A-frame level in terms of accuracy if not of portability, although the dioptra would be superior in both respects. Certainly there is no evidence whatsoever that the chorobates found any use at all on the gently graded aqueducts of the Roman West. In short, with its cumbrous bulk and its absence of sights and staff, it does not deserve to be considered a serious surveyor's level.

[82] Lewis 1999b.

[83] Comparable examples of inventors' puffs are ***Dioptra*** **1**, **33** and **34** where Hero dismisses existing dioptras, gromas and hodometers, and Philo, *Artillery* 59–67 on Philo's supposed improvements to catapult design.